XAM 3A: FIND $X(t)$ FOR $X'' + 4X' + 6X = 10 \cos 2t$

USE METHOD OF UNDETERMINED COEFFICIENTS. LET $X_{TRIAL}(X_T) = A\cos 2t + B \sin 2t$.

$X_T = A\cos 2t + B\sin 2t \implies 6X_T = 6A\cos 2t + 6B\sin 2t$

$X_T' = -2A\sin 2t + 2B\cos 2t \implies 4X_T' = -8A\sin 2t + 8B\cos 2t$

$X_T'' = -4A\cos 2t - 4B\sin 2t \implies 1X_T'' = -4A\cos 2t - 4B\sin 2t$

COLLECT THE RESP. A,B sin/cos TERMS: $\implies X_T'' + 4X_T' + 6X_T = 2A\cos 2t - 8A\sin 2t + 14B\cos 2t - 4B\sin 2t$

$X'' + 4X' + 6X = 10\cos 2t \implies \underline{2A}\cos 2t \underline{-8A}\sin 2t + \underline{14B}\cos 2t \underline{-4B}\sin 2t = \underline{10}\cos 2t + \underline{0}\sin 2t$

$$\implies \begin{aligned} 2A + 14B &= 10 \\ -8A - 4B &= 0 \end{aligned}$$

SET SYSTEM: $\left(\begin{array}{cc|c} 2 & 14 & 10 \\ -8 & -4 & 0 \end{array}\right)$ COMBO(1,2,4)

$\left(\begin{array}{cc|c} 2 & 14 & 10 \\ 0 & 52 & 40 \end{array}\right)$ MULT(1, $\frac{1}{2}$) MULT(2, $\frac{1}{52}$)

$\left(\begin{array}{cc|c} 1 & 7 & 5 \\ 0 & 1 & \frac{10}{13} \end{array}\right)$ COMBO(2,1,-7)

$\left(\begin{array}{cc|c} 1 & 0 & -\frac{5}{13} \\ 0 & 1 & \frac{10}{13} \end{array}\right) \implies A = -\frac{5}{13}$, $B = \frac{10}{13}$

PLUG A,B INTO → TO GET

$-\frac{10}{13} + \frac{140}{13} = 10$ ✓

$\frac{40}{13} - \frac{40}{13} = 0$ ✓

SO, $x(t) = \frac{-5}{13}\cos 2t + \frac{10}{13}\sin 2t$

B: FIND X_P FOR $y'' + 3y' + 2y = xe^{2x}$ USING VARIATION OF PARAMETERS

FIND SOLUTION (HOMOGENEOUS): $y'' + 3y' + 2y = 0 \implies y_H = C_1 e^{-2x} + C_2 e^{-x}$

$W(x) = \begin{vmatrix} e^{-2x} & e^{-x} \\ -2e^{-2x} & -e^{-x} \end{vmatrix} = e^{-3x}$

$y_1 = e^{-2x}$, $y_2 = e^{-x}$, $W(x) = e^{-3x}$

$f(x) = xe^{2x}$

USE FORMULA (33): $y_P = -y_1 \int \frac{y_2(x) f(x)}{W(x)} dx + y_2 \int \frac{y_1(x) f(x)}{W(x)} dx$

$$= -e^{-2x} \int \frac{(e^{-x}) x e^{2x}}{e^{-3x}} dx + e^{-x} \int \frac{(e^{-2x}) x e^{2x}}{e^{-3x}} dx \equiv -e^{-2x} \int x e^{4x} dx + e^{-x} \int x e^{3x} dx$$

$$= -e^{-2x}\left(\frac{1}{16}\left(4xe^{4x} - e^{4x}\right)\right) + e^{-x}\left(\frac{1}{9}\left(3xe^{3x} - e^{3x}\right)\right)$$

NOTE THAT THE e^{-2x} AND e^{-x} WILL CAUSE EVERYTHING TO HAVE A FACTOR OF e^{2x}

$$= -\frac{1}{4}xe^{2x} + \frac{1}{16}e^{2x} + \frac{1}{3}xe^{2x} - \frac{1}{9}e^{2x}$$ COMBINING LIKE TERMS

$$y_P = \frac{1}{12}xe^{2x} - \frac{7}{144}e^{2x}$$

WE MAY CHECK y_P BY TAKING y_P, y_P', y_P'' AND INSERTING INTO $y'' + 3y' + 2y$ AND VERIFYING THE RESULT IS xe^{2x}.