Name _____

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Differential Equations 2280
Midterm Exam 1 [8:35]
Wednesday, 25 February 2009

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Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Quadrature Equations)**

    (a) [25%] Solve $y' = \dfrac{3 + x^2}{1 + x^2}$.

    (b) [25%] Solve $y' = (2\sin x + \cos x)(\sin x - 2\cos x)$.

    (c) [25%] Solve $y' = \dfrac{x \tan\left(\ln(1 + x^2)\right)}{1 + x^2}$, $y(0) = 2$.

    (d) [25%] Find the position $x(t)$ from the velocity model $\dfrac{d}{dt}(t^2 v(t)) = 0$, $v(2) = 10$ and the position model $\dfrac{dx}{dt} = v(t)$, $x(2) = -20$.

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2. **(Classification of Equations)**
   The differential equation $y' = f(x, y)$ is defined to be **separable** provided $f(x, y) = F(x)G(y)$ for some functions F and G.

   (a) [40%] Check ($\boxed{X}$) the problems that can be put into separable form. No details expected.

$\square$ $y' + xy = y(2x + e^x) + x^2 y$	$\square$ $y' = (x-1)(y+1) + (1-x)y$
$\square$ $y' = 2e^{2x-y}e^{3y} + 3e^{3x+2y}$	$\square$ $y' + x^2 e^y = xy$

   (b) [10%] Is $y' + x(y+1) = ye^x + x$ separable? No details expected.

   (c) [10%] Give an example of $y' = f(x, y)$ which is separable and linear but not quadrature. No details expected.

   (d) [40%] Apply tests to show that $y' = x + e^y$ is not separable and not linear. Supply all details.

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3. **(Solve a Separable Equation)**
   Given $(x + 3)(y + 1)y' = \left((x + 3)e^{-x+2} + 3x^2 + 2\right)(y - 1)(y + 2)$.

   Find a non-equilibrium solution in implicit form.
   To save time, **do not solve** for y explicitly and **do not solve** for equilibrium solutions.

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4. **(Linear Equations)**

   (a) [60%] Solve the linear model $5x'(t) = -160 + \dfrac{25}{2t+3}x(t)$, $x(0) = 32$. Show all integrating factor steps.

   (b) [20%] Solve the homogeneous equation $\dfrac{dy}{dx} - (2x)y = 0$.
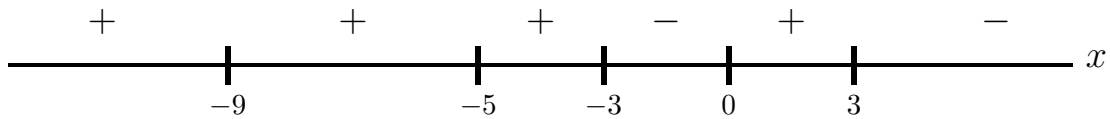
   (c) [20%] Solve $5\dfrac{dy}{dx} + 10y = 7$ using the superposition principle $y = y_h + y_p$. Expected are answers for y_h and y_p.

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5. **(Stability)**
   (a) [50%] Draw a phase line diagram for the differential equation

   $$\frac{dx}{dt} = \left(\ln(1 + 5x^2)\right)^{1/5} (|2x - 1| - 3)^3 (2 + x)^2 (4 - x^2)(1 - x^2)^3 e^{\cos x}.$$

   Expected in the phase line diagram are equilibrium points and signs of dx/dt.

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   (b) [50%] Assume an autonomous equation $x'(t) = f(x(t))$. Draw a phase diagram with at least 12 threaded curves, using the phase line diagram given below. Add these labels as appropriate: funnel, spout, node [neither spout nor funnel], stable, unstable.

   ```
        +            +         +     −     +          −
   ─────────┼──────────┼─────┼─────┼──────┼──────────── x
           −9         −5    −3    0      3
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