Rouben Azad, Ryanne wiser

March 30, 2012

Professor Gustafson

MATH 2270

Picture of Linear Algebra

Cameras are used daily to capture images and store them as data for later use. We've all had contact with this machine and as small as it might get, the technology behind it is brilliant. There are many factors involved in it; from the use of the LCD panels, the lenses or the sensors. But we will be discussing the use of color correction.

The process of recording this data is started by the use of light striking the sensor and causing various cells to react, which then is recorded and converted into a matrix of numbers. This matrix of numbers, along with the algorithms and the use of linear transformations create the application of linear algebra called color correction.

Color correction is basically a filter that alters the overall color of the light. Without this filter an image may have a mix of colors. Applying the color correction gels, filters that go in front of the lens or light source, can change the color of the light to match the source. If you have used a camera device in different lighting conditions, like in the sun, in the shade, or at night, you would notice a distinct color cast depending on the source of the light. We will

BEFORE   AFTER

further analyze how this works and the features involved in it and how it all comes together.

Color temperature is also used to amplify the resulting image. This is a body, called a "black body" in physics that emits light when it is heated. The color depends on the temperature; for example, a metal bar heated will turn red, then yellow and then blue-white. The hotter the body get the more the color will move from red towards blue, and that is the same idea behind the RGB space.
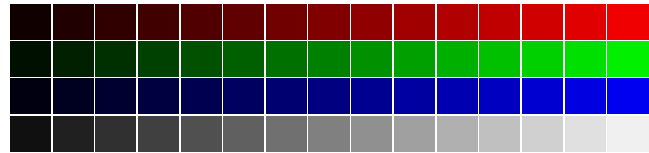
RGB (red, green, blue) space is the color balance, where we assume which colors are the most sensitive. Red, green, and blue are traditionally the primary colors used in almost machines that portray images. To convey the right color, you need to mix the RGB intensity just like the blocks above; in the red row each block towards the right increases the intensity of red, same with the others. Therefore, by mixing and matching each primary color you will get a unique color.

Let's say we wish to obtain the color yellow. We will define 3 vectors I, J and K, which are the primary colors red, green, blue.

$$i = \text{red} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad j = \text{green} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad k = \text{blue} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{yellow} = i + j = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Red and green will make yellow in the RGB space; simply add the vectors I and J as seen above. Any color in the RGB space can be made as a linear combination of the vectors I, J and K.

RGB space can be defined mathematically as the XYZ space. There is a conversion step from the xy chromaticity of red, blue, green, and white in RGB space, which is linear, to the

XYZ space. Since this has only two-dimensions, x and y, there is an assumption that the white, or the luminance is consistent from the RGB space to the XYZ space. This is represented by the Y in the XYZ space.

The XYZ space is the algorithm where matrices of numbers that represent the red, green, and blue are used to store data or use the data in any way necessary for programming. RGB space and XYZ space are both vector spaces, which define a basis of one color space into another.

To convert from RGB space into XYZ space, take the equation z=1-x-y and plug it into the matrices for red, green, blue, and white.

$$r_{xyz} = \begin{bmatrix} r_x \\ r_y \\ 1-r_x-r_y \end{bmatrix}$$

$$g_{xyz} = \begin{bmatrix} g_x \\ g_y \\ 1-g_x-g_y \end{bmatrix} \qquad b_{xyz} = \begin{bmatrix} b_x \\ b_y \\ 1-b_x-b_y \end{bmatrix} \qquad w_{xyz} = \begin{bmatrix} w_x \\ w_y \\ 1-w_x-w_y \end{bmatrix}$$

Then solve for the transformation matrix by listing and defining the x and y and z by themselves.

$$x = \frac{X}{X+Y+Z}, \; y = \frac{Y}{X+Y+Z} \text{ and } z = \frac{Z}{X+Y+Z}$$

And the vectors as:

$$v_{xyz} = \left(\frac{v_X}{v_X+v_Y+v_Z}, \frac{v_Y}{v_X+v_Y+v_Z}, \frac{v_X}{v_Z+v_Y+v_Z} \right)$$

$$v_{xyz} = \frac{1}{v_X+v_Y+v_Z}(v_X, v_Y, v_X) = \frac{1}{v_X+v_Y+v_Z}v_{XYZ}$$

$$v_{XYZ} = (v_X+v_Y+v_Z)v_{xyz}$$

Because white is the maximum luminance, which is usually set as a value of either 1 or 100, we can create an equation for w as well.

$$w_y = \frac{w_Y}{w_X + w_Y + w_Z}$$

$$(w_X + w_Y + w_Z) = \frac{w_Y}{w_y} = \frac{1}{w_y}$$

$$w_{XYZ} = (w_X + w_Y + w_Z)w_{xyz} = \left(\frac{1}{w_y}\right)w_{xyz}$$

This leads to the transformation matrix M:

$$M = \begin{bmatrix} (r_X + r_Y + r_Z)r_x & (g_X + g_Y + g_Z)g_x & (b_X + b_Y + b_Z)b_x \\ (r_X + r_Y + r_Z)r_y & (g_X + g_Y + g_Z)g_y & (b_X + b_Y + b_Z)b_y \\ (r_X + r_Y + r_Z)r_z & (g_X + g_Y + g_Z)g_z & (b_X + b_Y + b_Z)b_z \end{bmatrix}$$

$$= \begin{bmatrix} r_x & g_x & b_x \\ r_y & g_y & b_y \\ r_z & g_z & b_z \end{bmatrix} \begin{bmatrix} (r_X + r_Y + r_Z) & 0 & 0 \\ 0 & (g_X + g_Y + g_Z) & 0 \\ 0 & 0 & (b_X + b_Y + b_Z) \end{bmatrix}$$

This is where the white values come into play. They help define the three unknowns left to convert from RGB to XYZ space.

$$w_{XYZ} = M w_{RGB}$$

$$w_{XYZ} = \begin{bmatrix} r_x & g_x & b_x \\ r_y & g_y & b_y \\ r_z & g_z & b_z \end{bmatrix} \begin{bmatrix} (r_X + r_Y + r_Z) & 0 & 0 \\ 0 & (g_X + g_Y + g_Z) & 0 \\ 0 & 0 & (b_X + b_Y + b_Z) \end{bmatrix} w_{RGB}$$

Color compression and RGB space are just the beginning of the camera's applications of linear algebra. The mathematics only become more complicated from here and continues on into topics such as image compression. The uses of linear algebra to create, fix, and alter pictures are endless.

Work Cited

Allan Engelhardt. Travel and location photography. Color Temperature and Color Correction in

Photography. Retrieved from: http://www.aeimages.com/learn/color-correction.html

John Jackman (2004). Lighting for Digital Video and Television (second edition ed.). Focal

Press. Retrieved from: http://en.wikipedia.org/wiki/Color_correction

Ryan Juckett, RGB Color Space Conversion. Retrieved from:

http://www.ryanjuckett.com/programming/graphics/27-rgb-color-space-convers

ion?showall=1