Find an LU-decomposition for the matrix A.

Means: Find a frame sequence for A using only combo and mult operations. Don't use any combo that changes upper triangle entries. Stop when the frame is first upper triangular, Then

$$A_m = E_m \cdots E_2 E_1 A$$

or

$$A = E_1^{-1} E_2^{-1} \cdots E_m^{-1} A_m$$

$= (\text{lower triang})(\text{upper triang})$

$= LU$

Example: $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 1 & 4 & 3 \end{pmatrix}$

$A_1 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 1 & 4 & 3 \end{pmatrix}$

$A_2 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 2 & 0 \end{pmatrix}$ combo(1,3,−1) $\qquad E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

$A_3 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & -4 \end{pmatrix}$ combo(2,3,−1) $\qquad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

= upper triangular matrix U

$U = A_3$

$= E_2 E_1 A_1$

$A = A_1$

$A = E_1^{-1} E_2^{-1} U$

$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & -4 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & -4 \end{pmatrix}$

$= LU$