Strang: Chapter 7

Section 7.1. Exercises 1, 3, 7, 8, 10, 11

Section 7.2. Exercises 1, 2, 5, 10, 14, 20, 21, 26

Section 7.3. Recommended singular value decomposition problems: Exercises 4, 7, 21, 23

Some Answers

7.1. Exercises 3, 7, 10 have textbook answers.

7.1-1. With $w = 0$ linearity gives $T(v + 0) = T(v) + T(0)$. Thus $T(0) = 0$. With $c = -1$ linearity gives $T(-\vec{0}) = -T(\vec{0})$. This is a second proof that $T(0) = 0$.

7.1-8. (a) The range of $T(v_1, v_2) = (v_1 - v_2, 0)$ is the line of vectors $(c, 0)$. The nullspace is the line of vectors (c, c). (b) $T(v_1, v_2, v_3) = (v_1, v_2)$ has Range $\mathcal{R}^2$, kernel $\{(0, 0, v_3)\}$ (c) $T(0) = 0$ has Range $\{0\}$, kernel $\mathcal{R}^2$ (d) $T(v_1, v_2) = (v_1, v_1)$ has Range = multiples of $(1, 1)$, kernel = multiples of $(1, -1)$.

7.1-11. For multiplication $T(v) = Av$: $V = \mathcal{R}^n$, $W = \mathcal{R}^m$; the outputs fill the column space; v is in the kernel if $Av = 0$.

7.2. Exercises 5, 14, 20 have textbook answers.

7.2-1. For $Sv = d^2v/dx^2$, $v_1, v_2, v_3, v_4 = 1, x, x^2, x^3$, $Sv_1 = Sv_2 = 0$, $Sv_3 = 2v_1$, $Sv_4 = 6v_2$. The matrix for S is $B = \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

7.2-2. $Sv = d^2v/dx^2 = 0$ for linear functions $v(x) = a + bx$. All $(a, b, 0, 0)$ are in the nullspace of the second derivative matrix B.

7.2-10. The matrix for T is $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. For the output $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ choose input $v = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. This means: For the output w_1 choose the input $v_1 - v_2$.

7.2-21. Basis w to basis v: $\begin{pmatrix} 0.0 & 1 & 0 \\ 0.5 & 0 & -0.5 \\ 0.5 & -1 & 0.5 \end{pmatrix}$. Basis v to basis w: inverse matrix $= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \end{pmatrix}$. The key idea: The matrix multiplies the coordinates in the v basis to give the coordinates in the w basis.

7.2-26. Start from $A = LU$. Row 2 of A is ℓ_{21}(row 1 of U)+ ℓ_{22} (row 2 of U). The change of basis matrix is always invertible, because basis goes to basis.

7.3: Problems 4, 7, 23 have answers in Strang's book.

7.3-21. Column times row multiplication gives $A = U\Sigma V^T = \sum \sigma_i \vec{u}_i \vec{v}_i^T$ and also $A^+ = V\Sigma^+ U^T = \sum(1/\sigma_i)\vec{v}_i\vec{u}_i^T$. Multiplying A^+A and using orthogonality of each $\vec{u}_i$ to all other $\vec{u}_j$ gives the projection matrix $A^+A = \sum(1)\vec{v}_i\vec{v}_i^T$. Similarly $AA^+ = \sum(1)\vec{u}_i\vec{u}_i^T$ from $VV^T = I$.