Math 3150 Partial Differential Equations for Engineers Spring 2006

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1. Consider separately each of the following equations for the function $u(x,t)$. Does the superposition principle hold for it?
   (a) $u_{tt} = u_{xx} + 1$
   (b) $u_{tt} = u_{xx} + x$
   (c) $u_{tt} = u_{xx} + t$
   (d) $u_{tt} = u_{xx} + u$
   (e) $u_{tt} = u_{xx} + u^2$

2. Consider the following equation for the function $u(x, y)$: $xu_x + yu_y = 0$.
On which curves does the function $u(x, y)$ stay constant? Sketch those curves.

3. Consdider function $f(x)$ which on the interval $-p < x < p$ (p is a positive number) is defined as

$$f(x) = \begin{cases} 1 \text{ if } |x| < p/2, \\ 0 \text{ if } -p < x < -p/2, \\ 0 \text{ if } p/2 < x < p. \end{cases}$$

(a) Assume that this function is $2p$-periodic.
Sketch its graph over 3 periods and find its Fourier series.

(b) Assume that $f(x) \equiv 0$ outside the interval $-p < x < p$.
Sketch its graph and find its Fourier transform. Sketch the Fourier transform.

4. Solve the following boundary value problem for the function $u(x,t)$:

$$\begin{aligned} u_{tt} &= u_{xx} \\ u(0,t) &= u(\pi,t) = 0 \\ u(x,0) &= \sin(x) + 0.05\sin(7x) \\ u_t(x,0) &= 0.1\sin(3x) \end{aligned}$$

5. Solve the following boundary value problem for the function $u(x,t)$:

$$\begin{aligned} u_t &= u_{xx} \\ u(0,t) = u(\pi,t) &= 0 \\ u(x,0) &= 1 \end{aligned}$$

6. Find the steady temperature in a bar of length L if the ends of the bar are held at the temeratures T_1 and T_2 (the latteral sides of the bar are insulated).

7. Solve the following boundary value problem for the function $u(x, y, t)$:

$$\begin{aligned} u_{tt} &= u_{xx} + u_{yy}, \\ u(x, 0, t) = u(0, y, t) = u(x, \pi, t) &= u(\pi, y, t) = 0, \\ u(x, y, 0) = 0,\ u_t(x, y, 0) &= (\sin x)(\sin 3y). \end{aligned}$$

8. Solve the following boundary value problem for the function $u(x, y)$:

$$u_{xx} + u_{yy} = 0 \quad \text{in the square} \quad 0 < x < \pi, \quad 0 < y < \pi,$$
$$u(x, 0) = 0, \; u(0, y) = 0, \; u(x, \pi) = \sin x, \; u(\pi, y) = \sin y.$$