Math 3150-1, Practice Midterm Exam 1

September 18 2008

Total points: 100/100.

Problem 1 (20 pts) Consider the $2p-$periodic function

$$f(x) = \begin{cases} x - \frac{p}{2} & \text{if } 0 \le x \le p \\ x + \frac{p}{2} & \text{if } -p \le x \le 0 \end{cases}$$

(a) Sketch $f(x)$ for $x \in [-p, 3p]$. Carefully label your axis.

(b) Is f continuous? Piecewise continuous? Piecewise smooth?

(c) Find the Fourier series of f.

Problem 2 (15 pts) Decide whether the following partial differential equations are linear or non-linear and if linear, whether they are homogeneous or non-homogeneous. Determine the order of the differential equation.

(a) $\begin{cases} u_{xxxx} + u_{xx} = u_{tt} \\ u(0,t) = u_x(0,t) = 0 \\ u(1,t) = u_x(1,t) = 0 \end{cases}$

(b) $u_t = uu_x + u_{xx}$

(c) $u_{xx} + u_{yy} = \exp[-x^2 - y^2]$

(d) $\begin{cases} u_{xx} = u_t \\ u(0,t) = 0 \\ u_x(1,t) + u(1,t) = 0 \end{cases}$

Problem 3 (25 pts) Let $f(x) = x(1-x)$ be defined on $[0,1]$.

(a) Sketch the odd and even $2-$periodic extensions of $f(x)$. Carefully label your axis.

(b) Calculate the Sine and Cosine Series expansions of $f(x)$.

(c) Solve the one dimensional wave equation

$$\begin{cases} u_{tt} = 2u_{xx} \\ u(0,t) = u(1,t) = 0 \\ u(x,0) = f(x) \\ u_t(x,0) = 0 \end{cases}$$

(d) Solve the one dimensional wave equation

$$\begin{cases} u_{tt} = 2u_{xx} \\ u(0,t) = u(1,t) = 0 \\ u(x,0) = 0 \\ u_t(x,0) = f(x) \end{cases}$$

Problem 4 (20 pts) Consider the one dimensional wave equation

$$\begin{cases} u_{tt} = u_{xx} \\ u(0,t) = u(1,t) = 0 \\ u(x,0) = 0 \\ u_t(x,0) = \cos(\pi x) \end{cases}$$

Solve this differential equation using

(a) Fourier series

(b) D'Alembert's method

Problem 5 (20 pts) Consider the following one dimensional heat equations:

$$\text{(A)} \begin{cases} u_t = u_{xx} \\ u(0,t) = u(1,t) = 0 \\ u(x,0) = x(1-x). \end{cases} \qquad \text{(B)} \begin{cases} u_t = u_{xx} \\ u(0,t) = 1, u(1,t) = 0 \\ u(x,0) = 1 - x^2. \end{cases}$$

(a) Give a physical interpretation of the boundary conditions for (A) and (B).

(b) Solve problem (A). **Hint:** You may use the results from problem 3c.

(c) What is the steady state temperature distribution $s(x)$ for (B)?

(d) Use your answer to (a) together with $s(x)$ to solve problem (B).