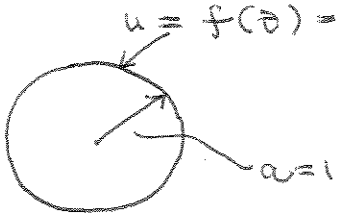
Examples for 4.4

Solve the Laplace's equation problems in following circular regions

1



$$u(r,\theta) = a_0 + \sum_{n=1}^{\infty} \left(\frac{r}{a}\right)^n [a_n \cos n\theta + b_n \sin n\theta]$$

$$u(1,\theta) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\theta + b_n \sin n\theta]$$

$$= 3 \sin 5\theta$$

By direct comparison of both sides show

$$a_0 = 0$$

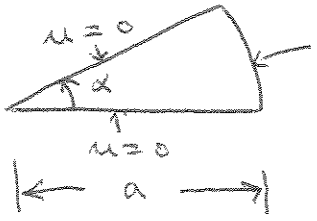
$$b_n = \begin{cases} 0 & n \neq 5 \\ 3 & n = 5 \end{cases}$$

$$a_n = 0 \qquad n = 1, 2, \cdots$$

The solution of the problem is

$$u(r,\theta) = 3 r^5 \sin 5\theta$$

2



$$u = \sum_{n=1}^{\infty} b_n \left(\frac{r}{a}\right)^{\left(\frac{n\pi}{\alpha}\right)} \sin \frac{n\pi}{\alpha}\theta$$

$$u(1,\theta) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{\alpha}\theta$$

$$= \sin \frac{4\pi}{\alpha}\theta$$

$$b_n = \begin{cases} 0 & n \neq 4 \\ 1 & n = 4 \end{cases}$$

So the solution is

$$u(r,\theta) = r^{\frac{4\pi}{\alpha}} \sin \frac{4\pi}{\alpha}\theta$$