Section 1.2

(2) (a) Since u and v satisfy $\frac{\partial u}{\partial t} = -\frac{\partial v}{\partial x}$ for $t \geq 0$

$$\frac{\partial u}{\partial t}\Big|_{t=0} = -\frac{\partial v}{\partial x}\Big|_{t=0} = -\frac{\partial}{\partial x}\left[v|_{t=0}\right] = -\frac{\partial}{\partial x} h(x)$$

$$= -h'(x)$$

$u|_{t=0} = f(x)$ is obvious

(b) $v|_{t=0} = h(x)$ is obvious from the given condition.

Since $\frac{\partial v}{\partial t} = -\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial t}\Big|_{t=0} = -\frac{\partial u}{\partial x}\Big|_{t=0} = -\frac{\partial}{\partial x}\left[u|_{t=0}\right]$

$$= -\frac{\partial}{\partial x}\left[f(x)\right] = -f'(x)$$

(16) $f(x) = \frac{1}{2}\sin\frac{\pi x}{L} + \frac{1}{4}\sin\frac{3\pi x}{L}$, $g(x) = 0$

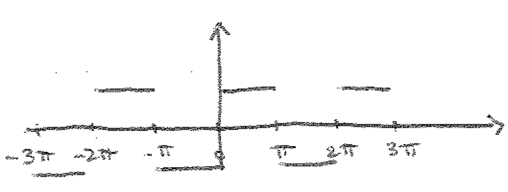
$$u(x,t) = \frac{1}{2}\sin\frac{\pi x}{L}\cos\frac{\pi c t}{L} + \frac{1}{4}\sin\frac{3\pi x}{L}\cos\frac{3\pi c t}{L}$$

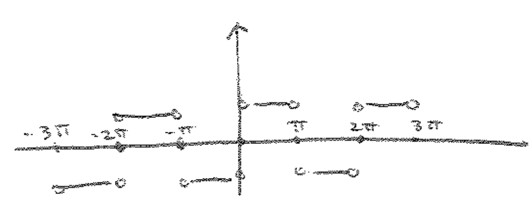
Section 2.1

(10) (a) π, (b) 4π (c) 2π (d) 2π

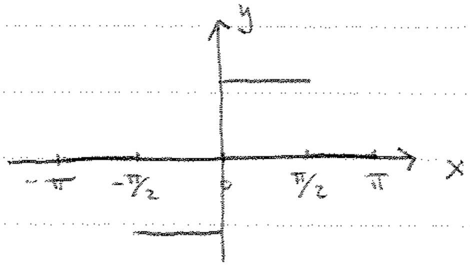
Section 2.2

(2)





(6)



$a_0 = 0$

$a_n = 0, \quad n = 1, 2, \dots$

$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} 1\cdot \sin nx\,dx + \int_{\pi/2}^{\pi} 0\cdot \sin nx\,dx\right] = \frac{2}{\pi}\left[-\frac{1}{n}\cos nx\Big|_0^{\pi/2}\right]$$

$$= \frac{2}{n\pi}\left[1 - \cos\frac{n\pi}{2}\right]$$

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(1-\cos\frac{n\pi}{2}\right)\sin nx \qquad x \neq \pm\frac{n\pi}{2}, \quad n = \overset{0}{1}, 2, \dots$$

(12) $f(x)$ is odd so $a_0 = 0$, $a_n = 0$, $n = 1, \dots$

$$b_n = \frac{2}{\pi}\int_0^{\pi}(\pi^2 x - x^3)\sin nx\,dx = 12\cdot\frac{(-1)^{n+1}}{n^3}$$

To calculate this integral, we need to use integration by parts ~~Section 3.3~~ three times

$$\int_0^{\pi}(\pi^2 x - x^3)\sin nx\,dx = -\frac{1}{n}\int_0^{\pi}(\pi^2 x - x^3)\,d(\cos nx)$$

$$= \cancelto{0}{-\frac{1}{n}(\pi^2 x - x^3)\cos nx\Big|_0^{\pi}} + \frac{1}{n}\int_0^{\pi}\cos nx\,(\pi^2 - 3x^2)\,dx$$

$$= \frac{1}{n^2}\int_0^{\pi}(\pi^2 - 3x^2)\,d(\sin nx)$$

$$= \cancelto{0}{\frac{1}{n^2}(\pi^2 - 3x^2)\sin nx\Big|_0^{\pi}} - \frac{1}{n^2}\int_0^{\pi}\sin nx\cdot(-6x)\,dx$$

$$= -\frac{6}{n^3}\int_0^{\pi} x\,d(\cos nx)$$

$$= -\frac{6}{n^3}\,x\cos nx\Big|_0^{\pi} + \cancelto{0}{\frac{6}{n^3}\int_0^{\pi}\cos nx\,dx}$$

$$= -\frac{6}{n^3}\pi\cos n\pi = -\frac{6}{n^3}\pi(-1)^n = \frac{6}{n^3}\pi\cdot(-1)^{n+1}$$

therefore

$$b_n = \frac{2}{\pi}\cdot\frac{6}{n^3}\pi(-1)^{n+1} = \frac{12}{n^3}(-1)^{n+1}$$