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Applied Differential Equations 2250-1
Midterm Exam 1
Wednesday, 16 February 2005

Instructions: This in-class ~~exam is 50 minutes.~~ ~~No calculators, notes, tables or~~ books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (Quadrature Equation)

Solve for $y(x)$ in the equation $y' = xe^{2x} - \cot x + \dfrac{x^3}{1+x^2}$.

$$\int y'\,dx = \int xe^{2x}dx - \int \cot x\,dx + \int \frac{x^3dx}{1+x^2}$$

$$y = \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}dx - \int \frac{\cos x\,dx}{\sin x} + \int \left(\frac{x^3+x}{x^2+1} + \frac{-x}{x^2+1}\right)dx$$

$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} - \ln|\sin x| + \frac{x^2}{2} - \frac{1}{2}\ln(x^2+1) + c$$

Scores were $70 \to 100$

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2. (Separable Equation Test)
The problem $y' = 3x - x^{3/5} - 2xy^3 + x^{3/5}y^3$ may or may not be separable. If it is, then write formulae for F, G and decompose the problem as $y' = F(x)G(y)$. Otherwise, explain in detail why it fails to be separable. Do not solve for y!

Let $x_0 = 1$, $y_0 = 0$, $f(x,y) = 3x - x^{3/5} - 2xy^3 + x^{3/5}y^3$

Define

$$F(x) = \frac{f(x,y_0)}{f(x_0,y_0)} = \frac{f(x,0)}{f(1,0)} = \frac{3x - x^{3/5}}{2}$$

$$G(y) = f(1,y) = 2 - 2y^3 + y^3 = 2 - y^3$$

Then

$$F(x)G(y) = \frac{3x - x^{3/5}}{2}(2 - y^3) = 3x - x^{3/5} - \frac{3x}{2}y^3 + \frac{x^{3/5}}{2}y^3 \neq 3x - x^{3/5} - 2xy^3 + x^{3/5}y^3 \quad (= f(x,y))$$

Then $FG \neq f$. The eq is not Separable.

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3. (Solve a Separable Equation)

Given $yy' = \dfrac{x^2+2x}{3+x}(1+2y^2)$, find the non-equilibrium solution in implicit form. Do not solve for y explicitly and do not find equilibrium solutions.

$$\frac{yy'}{1+2y^2} = \frac{x^2+2x}{3+x}$$ Separated form

$$\int \frac{yy'dx}{1+2y^2} = \int \frac{x^2+2X}{3+X}dx$$ method of quadr.

$$\int \frac{du}{4u} = \int \frac{x^2+2X}{3+X}dx$$ $u = 1+2y^2,\ du = 4yy'$

$$\begin{array}{r|l} & x \; -1 \\ \hline 3+x & x^2+2x \\ & \underline{x^2+\cancel{3x}} \\ & -x \\ & \underline{-x-3} \\ & 3 \end{array}$$

means $\dfrac{x^2+2X}{3+X} = X-1+\dfrac{3}{3+X}$

$$\frac{1}{4}\ln|u| = \int\left(x-1+\frac{3}{3+x}\right)dx$$

$$\boxed{\frac{1}{4}\ln|1+2y^2| = \frac{x^2}{2} - x + 3\ln(3+x) + c}$$

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4. (Linear Equations)

Solve (a) $3v'(t) = -30 - \dfrac{7}{t+2}v(t)$, $v(0) = 5$. Show all integrating factor steps.

(b) $y'(t) = v(t)$, $y(0) = 5$. Show all quadrature steps.

(a) $3v' + \frac{7}{t+2} v = -30$

$v' + \frac{7/3}{t+2} v = -10$

$\frac{(Qv)'}{Q} = -10$

$Q = e^{\int \frac{7/3\,dt}{t+2}}$
$= e^{\frac{7}{3}\ln|t+2|}$
$= |t+2|^{7/3}$
$= (t+2)^{7/3}$ near $t=0$.

$(Qv)' = -10Q$

$Qv = -10\int Q\,dt + c$

$v = \frac{-10}{Q}\int Q\,dt + \frac{c}{Q}$

$= \frac{-10}{(t+2)^{7/3}} \int (t+2)^{7/3} dt + \frac{c}{(t+2)^{7/3}}$

$= \frac{-10}{(t+2)^{7/3}} \frac{(t+2)^{10/3}}{10/3} + \frac{c}{(t+2)^{7/3}}$

$\boxed{v = -3(t+2) + c(t+2)^{-7/3}}$

$5 = -6 + c(2)^{-7/3}$

$\boxed{c = 11(2)^{7/3}}$

(b) $y(t) = \int_0^t v$

$= \int_0^t \left(-3(t+2) + c(t+2)^{-7/3}\right)dt$

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$\boxed{y = -\frac{3(t+2)^2}{2} + 6 + c\left[\frac{(t+2)^{-4/3} - 2^{-4/3}}{-4/3}\right]}$

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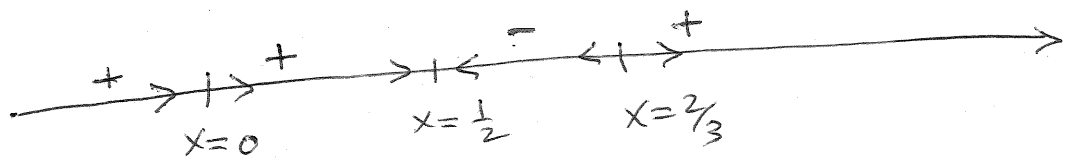
5. (Stability)
(a) Draw a phase line diagram for the chemical reaction equation $dx/dt = (2 - 3x)^3(1-2x)x^2$. Expected in the diagram are equilibrium points, signs of x' and flow direction markers (< and >).
(b) Draw a phase diagram using the phase line diagram of (a). Add these labels as appropriate: funnel, spout, node, stable, unstable.

(a) equilibria $= \frac{2}{3}, \frac{1}{2}, 0$

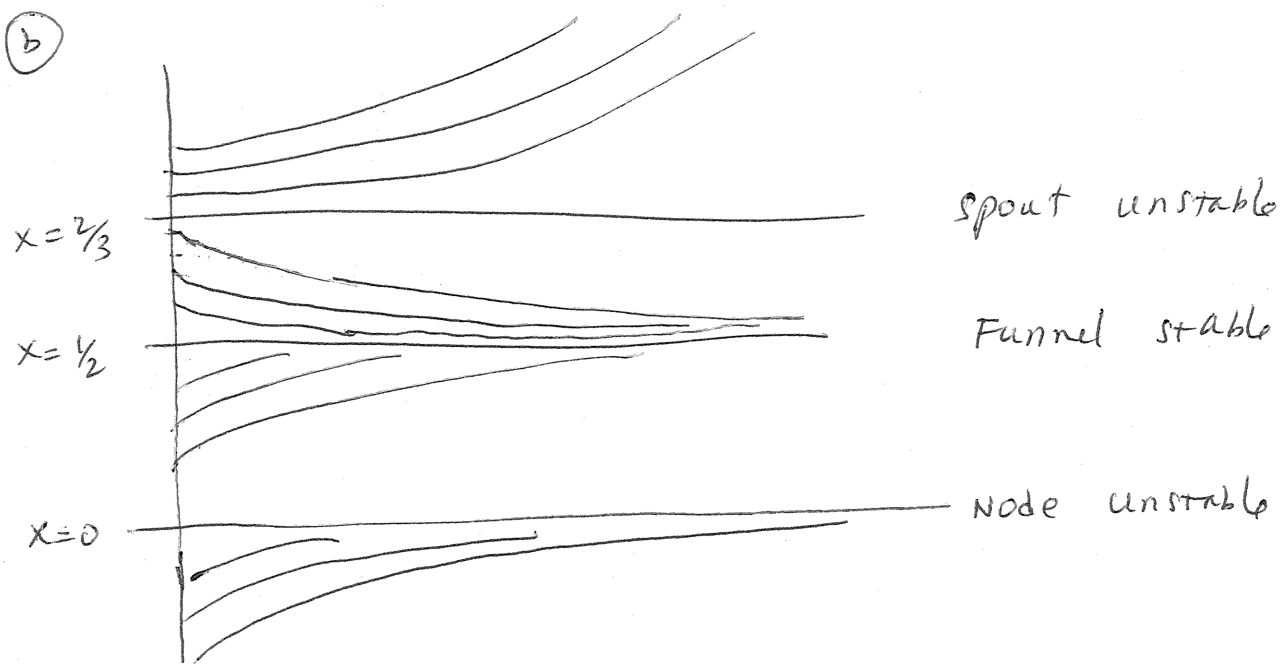
$f(x) = (2-3x)^3(1-2x)x^2$



$f(-1) = (+)(+)(+) = (+)$

$f(1) = (-)(-)(+) = +$

(b)



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