Mathematics 2210–1,2 PRACTICE EXAM I Fall 2019

1. Let $\vec{u} = (2, 2)$ and $\vec{v} = (3, -1)$. Find $\vec{u} + \vec{v}$ and illustrate this vector addition with a diagram in the plane, showing $\vec{u}$, $\vec{v}$ and the resultant vector. Illustrate multiplication by a scalar with a diagram showing $\vec{u}$, $3\vec{u}$, and $-\vec{u}$.

2. Consider the vectors $\vec{u} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\vec{v} = 3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$.

   (a) Find the length of $\vec{u}$.

   (b) Find $\vec{N} = \vec{u} \times \vec{v}$.

   (c) Find the cartesian equation of the plane with normal $\vec{N}$ through the point $P_0 = (1, 0, -1)$.

   (d) Find the vector projection of $\vec{v}$ onto $\vec{u}$.

3. Determine the area of the triangle with vertices $P = (0, 0), Q = (3, 2), R = (1, 4)$.

4. Consider $\vec{e_1} = (\sqrt{3}/2, 1/2)$ and $\vec{e_2} = (-1/2, \sqrt{3}/2)$. Show that $\vec{e_1}$ and $\vec{e_2}$ each have unit length and that they are orthogonal. Rewrite the vector $\vec{v} = 4\mathbf{i} + 5\mathbf{j}$ in the orthonormal basis $\vec{e_1} = (\sqrt{3}/2, 1/2), \vec{e_2} = (-1/2, \sqrt{3}/2)$. In other words, expand or write $\vec{v}$ as $\vec{v} = a\vec{e_1} + b\vec{e_2}$ where a and b are scalar values.

5. Let $\vec{u} = (4, 1)$ and $\vec{v} = (2, 3)$. Calculate $\vec{u} \times \vec{v}$. Use your result to find the angle θ between $\vec{u}$ and $\vec{v}$.

6. Find the work done by the force $\vec{F} = 6\mathbf{i} + 8\mathbf{j}$ pounds in moving an object from $(1, 0)$ to $(6, 8)$ where distance is in feet.

7. Find how much work you would do against the force of gravity ($\vec{F} = -mg\mathbf{j}$) to move an object of mass 5 kg from $(0, 0)$ to $(0, \sqrt{2})$, in units of meters. Do the same in moving it from $(0, 0)$ to $(1, 1)$, and compare your answer. How much work would you do in moving it from $(0, 0)$ to $(8, 0)$?

8. Given three points: $A = (0, 5, 3), B = (2, 7, 0), C = (-5, -3, 7)$

   (a) Which point is closest to the xz-plane? Explain your reasoning.

   (b) Which point lies on the xy-plane? Explain your reasoning.

9. Determine the equation of the plane spanned by the vectors:

$$\vec{u} = 1\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$$
$$\vec{v} = 2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$$

   and which contains the origin.

10. Find the curvature of the line parameterized by $\vec{r}(t) = (1, 1, 1) + (2, 3, 4)t$.

11. Find the arc length of the helix

$$\vec{r}(t) = a\sin(t)\mathbf{i} + a\cos(t)\mathbf{j} + ct\mathbf{k}$$

   for $0 \le t \le 2\pi$.

12. Find the equation of the plane orthogonal to the curve

$$\vec{r}(t) = (8t^2 - 4t + 3)\mathbf{i} + (\sin(t) - 4t)\mathbf{j} - \cos(t)\mathbf{k}$$

at the point $t = \pi/3$.

13. Determine the curvature κ of the helical curve parametrized by:

$$\vec{r}(t) = 7\sin(3t)\mathbf{i} + 7\cos(3t)\mathbf{j} + 14t\mathbf{k}$$

at $t = \pi/9$.

14. The acceleration of a particle's motion is

$$\vec{a}(t) = -9\cos(3t)\mathbf{i} + -9\sin(3t)\mathbf{j} + 2t\mathbf{k}.$$

The particle has initial velocity $\vec{v}_0 = \mathbf{i} + \mathbf{k}$ and intial position $\vec{x}_0 = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

(a) Determine the velocity function $\vec{v}(t)$.

(b) Determine the position function $\vec{x}(t)$.

15. Determine the position $\vec{r}(t) = (x(t), y(t))$ of a projectile fired from the point $(8, 3)$ with an initial speed of 20 f/s at an angle of 30°. **Be sure to show all your work**, not just the final formulas.