Solutions

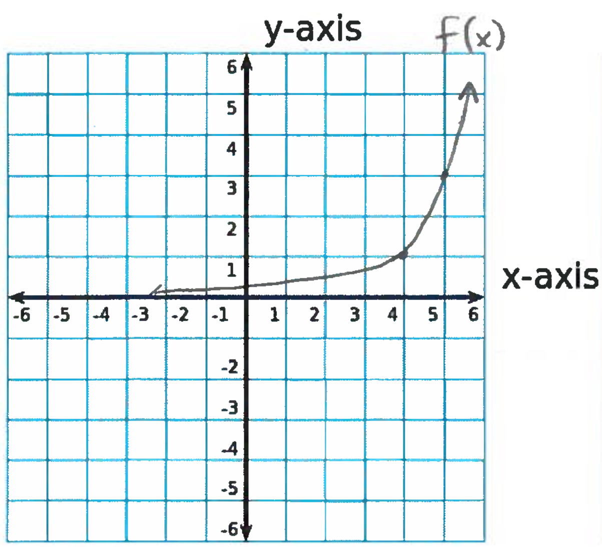
Math 1050-006 Midterm 3 Practice Test

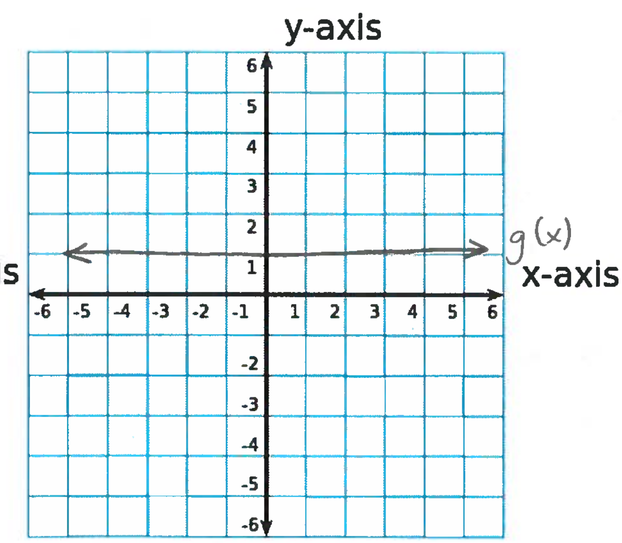
1.) Simplify the following exponential expressions:

a) $125^{\frac{-1}{3}} = \frac{1}{125^{\frac{1}{3}}} = \frac{1}{(5^3)^{\frac{1}{3}}} = \frac{1}{5^1} = \boxed{\frac{1}{5}}$

b) $(3^{200})^{\frac{1}{100}} = 3^{\frac{200}{100}} = 3^2 = \boxed{9}$

2.) Graph $f(x)=3^{(x-4)}$ and $g(x)=1^{(5x^2+3x-7)}$





exponential growth (base = 3)

y-int $= 3^{-4} = \frac{1}{81}$

$1^{5x^2+3x-7} = 1$ = constant function

3.) Benjamin Franklin before his death invested $1000 dollars into the Bank of Boston, if the value of his investment increased by 5% every year how much money would there be in his account 200 years later? (just set up your equation, you are not expected to be able to evaluate the answer)

17.2 million dollars (not taking into account inflation)

number of years

$$\underbrace{1000}_{\text{initial investment}}(1 + \underbrace{0.05}_{\text{growth rate}})^{200} = \boxed{1000(1.05)^{200}}$$

4.) What is the implied domain of the rational function $r(x)=\frac{3(x-5)^2(x^2+x+1)}{(x-2)(x-4)}$

Can't divide by zero $\Rightarrow x \neq 2, 4$

Domain $= \boxed{\mathbb{R} - \{2,4\}}$

5.) Identify the verical asymptotes, x-intercepts, and leading order term of the rational fucntion $r(x)=\frac{4(x+2)(x^2+1)}{3(x-3)(x-1)}$

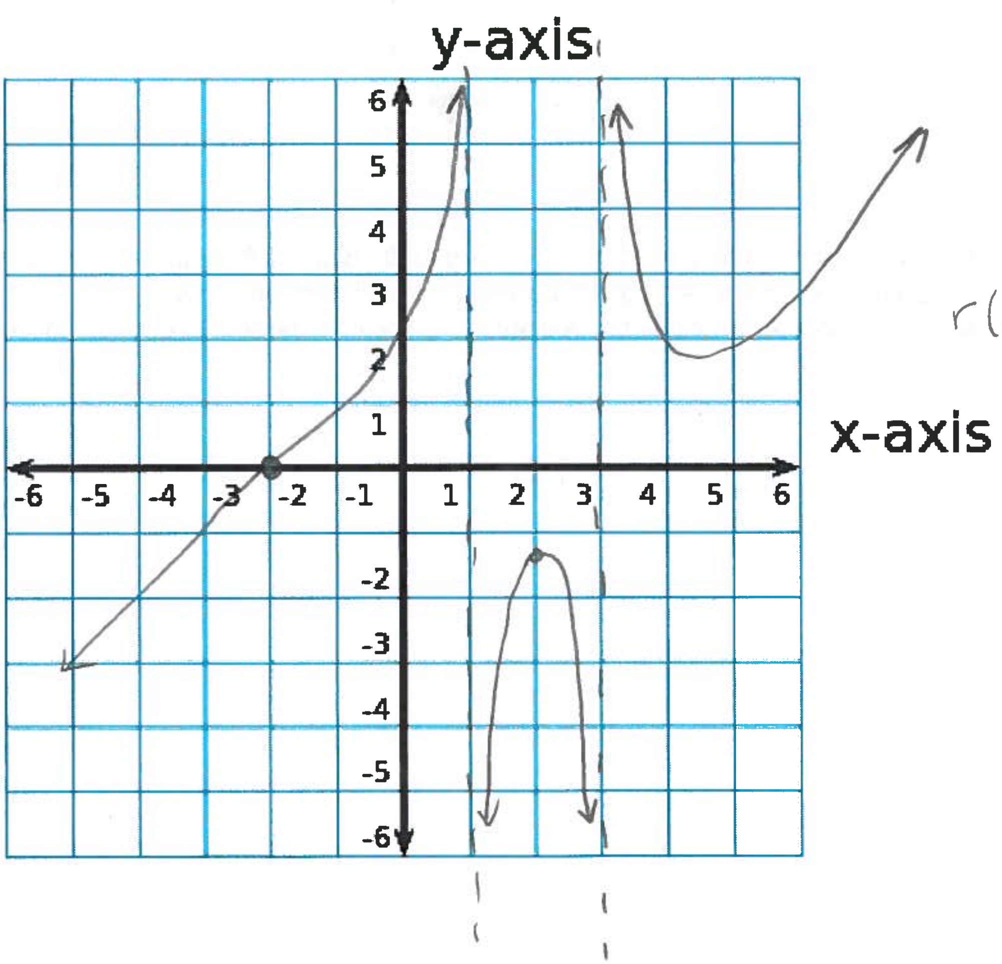
vertical asymptotes = roots of the denominator = 1, 3

x-intercepts = roots of the numerator = −2

$LOT = \frac{4(x)(x^2)}{3(x)(x)} = \frac{4}{3}x$

6.) Use your solutions from problem #6 to graph the rational function $r(x)=\frac{4(x+2)(x^2+1)}{3(x-3)(x-1)}$

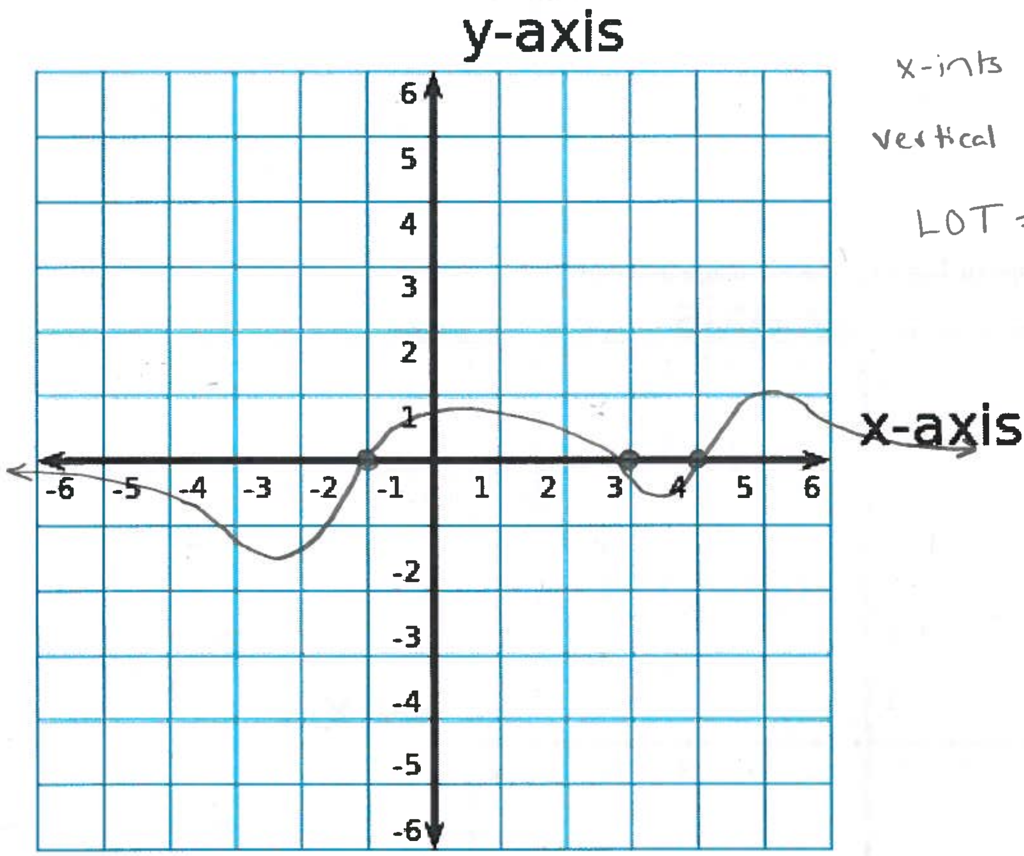
(from problem 5)



$r(2) = \frac{4(4)(5)}{3(-1)(1)} = \frac{-80}{3}$

$r(2) < 0$

7.) Graph the rational fucntion $r(x)=\frac{(x+1)(x-3)(x-4)(x^2+1)}{(x^2+x+1)(x^2+3)(x^2+20)}$



x-ints = $-1, 3, 4$

vertical asymptotes = none

$LOT = \frac{x^5}{x^6} = \frac{1}{x}$

8.) a) Solve for x if $3^x=13$

$\log_3(3^x) = \log_3(13) \Rightarrow x\log_3(3) = \log_3(13)$

$\boxed{x = \log_3(13)}$

b) Simplify $\log_5(125)$

$\log_5(125) = \log_5(5^3) = 3\log_5(5) = \boxed{3}$

9.) a) Simplify $\log_4(12)-\log_4(3)$

$\log_4(12) - \log_4(3) = \log_4\left(\frac{12}{3}\right) = \log_4(4) = \boxed{1}$

b) Simplify $\log_{10}(25)+\log_{10}(4)$

$\log_{10}(25) + \log_{10}(4) = \log_{10}(100) = \log_{10}(10^2) = 2\log_{10}(10)$

$= \boxed{2}$

10.) Use that $\log_a(x)$ and a^x are inverse functions to solve the equation: $\log_{36}(\frac{x}{2})+\log_{36}(2) = -\frac{1}{2}$

$$\log_{36}\left(\frac{x}{2}\right)+\log_{36}(2) = \frac{-1}{2}$$

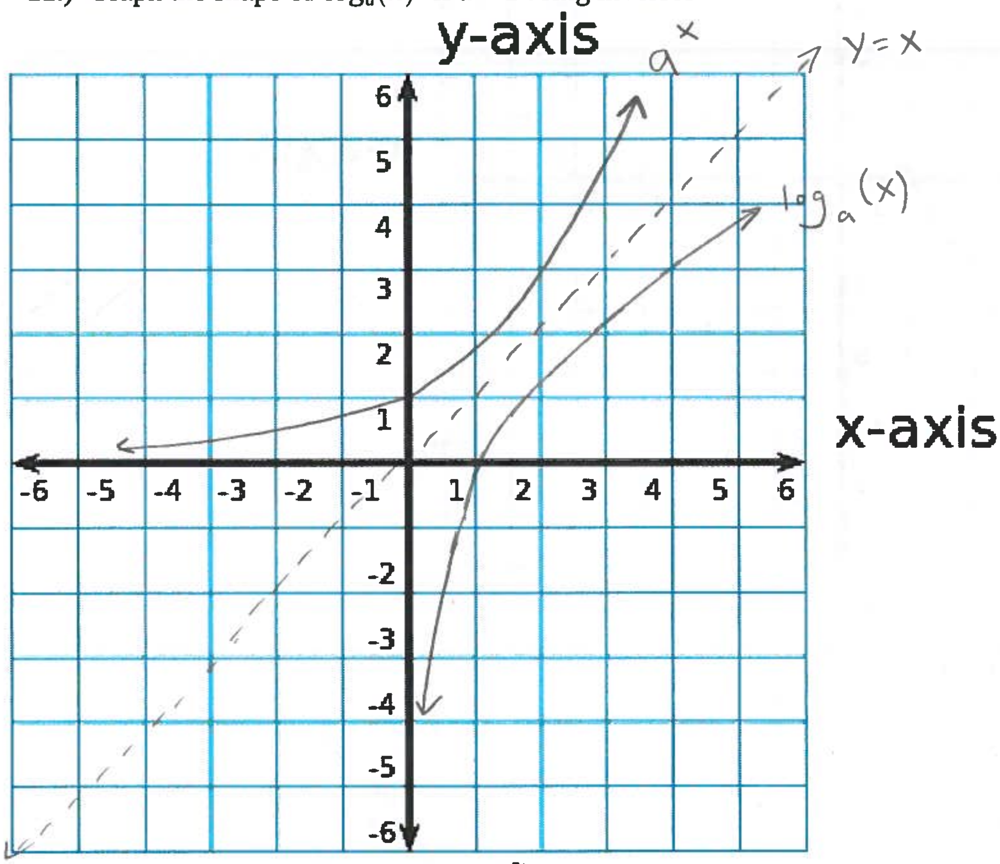
$$\log_{36}\left(\frac{x}{2}(2)\right) = \frac{-1}{2}$$

$$\log_{36}(x) = \frac{-1}{2}$$

$$36^{\log_{36}(x)} = 36^{\frac{-1}{2}}$$

$$x = \frac{1}{36^{1/2}} = \frac{1}{\sqrt{36}} = \boxed{\frac{1}{6}}$$

11.) Graph the shape of $\log_a(x)$ if a > 1 using inverses



12.) Solve the exponential equation: $(6^3)^x=27$ for x.

$$(6^3)^x = 27$$

$$6^{3x} = 27$$

$$\log_6(6^{3x}) = \log_6(27)$$

$$3x\log_6(6) = \log_6(27)$$

$$3x = \log_6(3^3) = 3\log_6(3)$$

$$\Rightarrow \quad 3x = 3\log_6(3)$$

$$\boxed{x = \log_6(3)}$$

13.) Solve the exponential equation: $e^{-x^2} = e^{x+5}e^{-11}$ for x

$$\frac{e^{-x^2}}{e^{x+5}e^{-11}} = 1$$

$$e^{-x^2-(x+5)-(-11)} = 1$$

$$e^{-x^2-x+6} = 1$$

$$\log_e(e^{-x^2-x+6}) = \log_e(1) = 0$$

$$(-x^2-x+6)\log_e(e) = 0$$

$$-x^2-x+6 = 0$$

$$x^2+x-6 = 0$$

$$(x+3)(x-2) = 0$$

$$\boxed{x = -3 \text{ or } 2}$$

14.) Solve the logarithmic function: $\log_e(\sqrt{x-4}) = 5$ for x

$$\log_e(\sqrt{x-4}) = 5$$

$$\log_e((x-4)^{1/2}) = 5$$

$$\frac{1}{2}\log_e(x-4) = 5$$

$$\log_e(x-4) = 10$$

$$e^{\log_e(x-4)} = e^{10}$$

$$x-4 = e^{10}$$

$$\boxed{x = e^{10}+4}$$

15.) Solve the logarithmic function: $\log_{10}((x+1)^{-5}) = -15$

$$\log_{10}((x+1)^{-5}) = -15$$

$$-5\log_{10}(x+1) = -15$$

$$\log_{10}(x+1) = 3$$

$$10^{\log_{10}(x+1)} = 10^3 = 1000$$

$$x+1 = 1000$$

$$\boxed{x = 999}$$

16.) Find the domain of g if $g(x) = \begin{cases} x^2 & \text{if } x \in (-\infty, 2) \\ 2x-4 & \text{if } x \in (3,4] \\ 4 & \text{if } x \in [4, \infty) \end{cases}$

Domain $= (-\infty, 2) \cup (3,4] \cup [4, \infty)$

$= (-\infty, 2) \cup (3, \infty)$

$= \mathbb{R} - [2,3]$

17.) Using g(x) from problem 1~~7~~6 evaluate the function if you are able to (if not write undefined and write why)

a) g(0) $\quad 0 \in (-\infty, 2) \Rightarrow g(x) = x^2$

$\Rightarrow g(0) = (0)^2 = \boxed{0}$

b) $g(\frac{5}{2})$ $\quad \frac{5}{2} \notin$ Domain

$\Rightarrow g(\frac{5}{2})$ is $\boxed{\text{undefined}}$

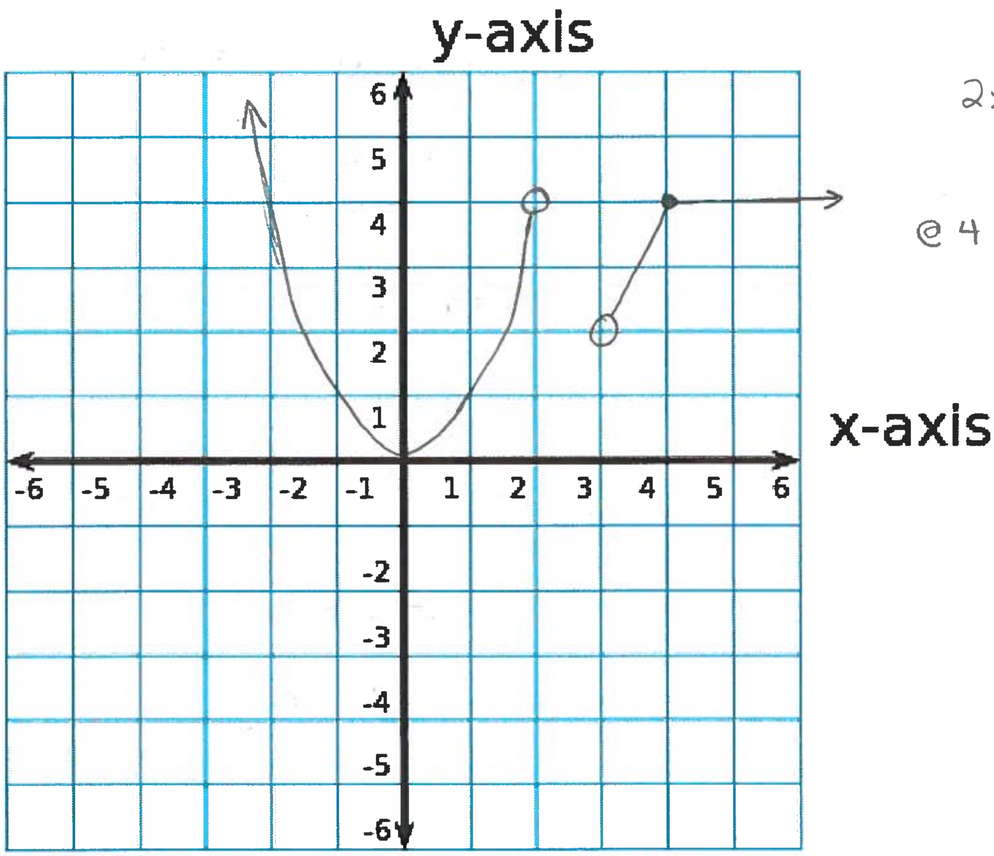
c) g(6)

$6 \in [4, \infty) \Rightarrow g(x) = 4$

$\Rightarrow g(6) = \boxed{4}$

18.) Graph the function g(x) from problem 17. $g(x)= \begin{cases} x^2 & \text{if } x\in(-\infty,2) \\ 2x-4 & \text{if } x\in(3,4] \\ 4 & \text{if } x\in[4,\infty) \end{cases}$

@ 2 $x^2=4$
not included

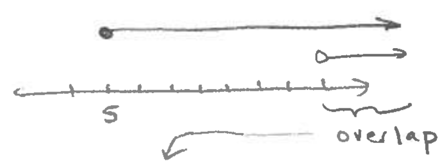


$2x-4$ @ 3
$2(3)-4=2$ not included

@ 4 $2x-4=2(4)-4=4$
included

19.) Solve for x if $|x-5| > 7$

$$|x-5| = \begin{cases} x-5 & \text{if } x \ge 5 \\ -x+5 & \text{if } x \le 5 \end{cases}$$



$\Rightarrow$ if $x \ge 5$ $|x-5| = x-5 > 7$

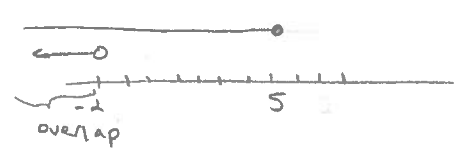
$x > 12$

$\Rightarrow$ $x > 12$

if $x \le 5$ $|x-5| = -x+5 > 7$

$-x > 2$

$x < -2$



$\Rightarrow$ $x < -2$

$\Rightarrow$ $\boxed{x > 12 \quad \text{or} \quad x < -2}$