Math 1050-006 Midterm 2 Practice Test Solutions

1.) Suppose $g(x) = x^2$, what is -4g(3x-7)+2

$-4(3x-7)^2+2=-4(9x^2-42x+49)+2=-36x^2+168x-194$

2.) Given that g(x) = x use graph transformations to graph f(x) = g(x-3) and h(x) = g(x)+2

The graph of h(x) is the graph of g(x) shifted up 2.
The graph of f(x) is the graph of g(x) shifted to the right 3.

3.) Given that $f(x) = x^2$, $f:[0,\infty)\rightarrow[0,\infty)$, both find and graph $f^{-1}(x)$.

$f^{-1}(x)=\sqrt{x}$ The graph of $f^{-1}(x)$ is the right half of the graph of x^2 flipped over the line y=x.

4.) Does $f(x) = x^2$, where f: $\mathbb{R}\rightarrow(-\infty,\infty)$ have an inverse? Justify why or why not using the ideas of onto and one-to-one.

$f(x)=x^2$ does not pass the horizontal line test so f(x) is not one-to-one.
Additionally the range of f(x) is $[0,\infty)$ and the target is $\mathbb{R}$ so as range $\neq$ target f(x) is not onto.
As f(x) is neither one-to-one nor onto it is not invertible.

5.) Find the inverse function for $h(x) = \dfrac{2x}{(5-3x)}$. *Assume the implied domain* $(x\neq\frac{5}{3})$

$h^{-1}(x)=\dfrac{5x}{3x+2}$

6.) If $g(x) = 3\sqrt[4]{x+5}$, find the implied domain of g(x).

The implied domain of g(x) is $[-5,\infty)$

7.) Solve for when $g(x) = 3\sqrt[4]{x+5} > 9$. i.e solve $3\sqrt[4]{x+5} > 9$ for x.

$\sqrt[4]{x+5} > 3 \quad \Rightarrow \quad x+5>3^4=81 \quad \Rightarrow \quad x>76$

8.) $f(x)=(x-7)-(x^2+4x+3)$, $g(x)=(x^2+3x^3-7x)$ solve for $h(x)$, if $h(x)=f(x)*g(x)$.
what is the degree and leading order term of $h(x)$?

$h(x)=-3x^5-10x^4-26x^3+11x^2+70x$
degree of h(x) = 5, leading order term = $-3x^5$

9.) Find the leading order term of $5(x-3)(x-5)(x-6)(x^2+1)(2x+x^2-7)$

Leading order term is $5(x)(x)(x)(x^2)(x^2) = 5x^7$

10.) Solve $\frac{10x^4-4x^3+5x-4}{x^2-3x}$ properly express your solution with a remainder if you find one.

$10x^2+26x+78+\frac{239x-4}{x^2-3x}$

11.) Given that the number 1 is a root of the polynomial $p(x)= 4x^4-3x^3+2x-3$ rewrite p(x) as the product of a linear and a cubic polynomial

$(x-1)(4x^3+x^2+x+3)$

12.) Give an upper and lower bound on the number of roots that the polynomial $p(x)= x^4-5x^5+3x^3-\pi$ has and justify your answer

The lower bound is 1, p(x) is degree 5, which is odd, and odd degree polynomials have at least 1 root
The upper bound is 5 as polynomials have at most the degree number of roots

13.) How many roots can a constant polynomial have? What about a linear polynomial?

A constant polynomial can have either 0 roots, or infinitely many roots.
A linear polynomial has exactly 1 root

14.) Find the slope, x-intercept, and y-intercept of the linear polynomial p(x)=3x-4

Slope = 3, x-intercept = $\frac{4}{3}$, y-intercept = -4

15.) Graph the linear polynomial $p(x)=-4x+3$

The graph of p(x) is the line connecting the points (0,3) and $(\frac{3}{4},0)$

16.) Rewrite the quadratic polynomial $p(x)=2x^2-3x-4$ in its completed square form using the completing the square formula: $p(x)=a(x+\frac{b}{2a})^2+c-\frac{b^2}{4a}$.

$2(x-\frac{3}{4})^2-\frac{41}{8}$

17.) Using your result from problem 16, graph p(x).

The graph of p(x) looks like the graph of $2x^2$ shifted to the right by $\frac{3}{4}$ and down by $\frac{41}{8}$.

18.) How many roots do we expect the quadratic polynomial $p(x)= -\frac{1}{2}x^2+3x-4$

Solve for these roots using the quadratic formula (if there are any).

Discriminant=b^2 - 4ac = 9-8 = 1 > 0 therefore we expect 2 roots.
The roots are 2, 4

19.) Completely factor $p(x)=2x^3-3x^2+4x-3$ How many roots does this polynomial have?

$2(x-1)(x^2-\frac{1}{2}x+\frac{3}{2})$ this polynomial has 2 roots

20.) Find the roots of $p(x)=-x^3+6x^2+7x$ and then use this information to graph the shape of the function.

p(x)= -x(x-7)(x+1) has roots 0,-1,7
the graph of this function is positive to the left of -1, negative between -1 and 0, positive between 0 and 7, and negative to the right of 7