FPP Geodesics

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University of Utah
October 20, 2018

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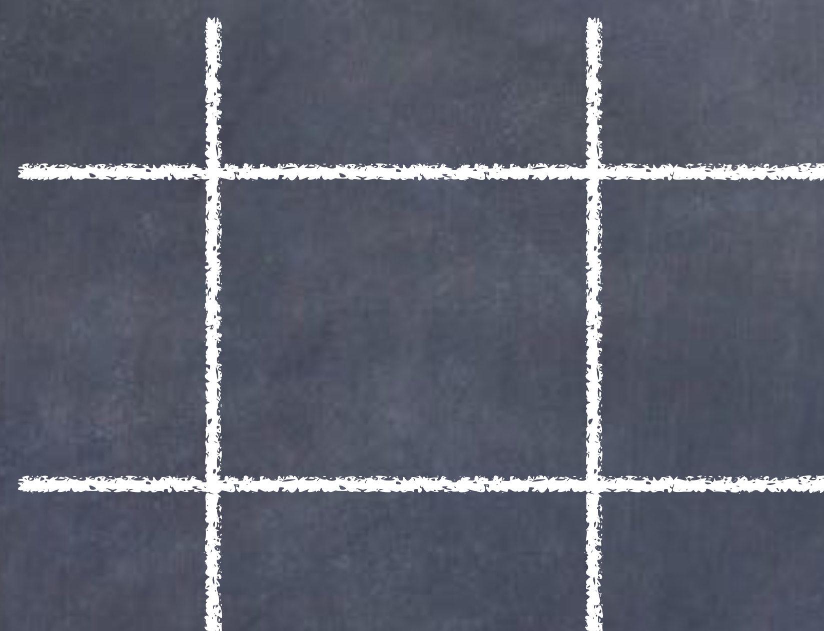
Joint with Arjun Krishnan (Rochester)
and Timo Seppäläinen (Madison)



$d \geq 2$

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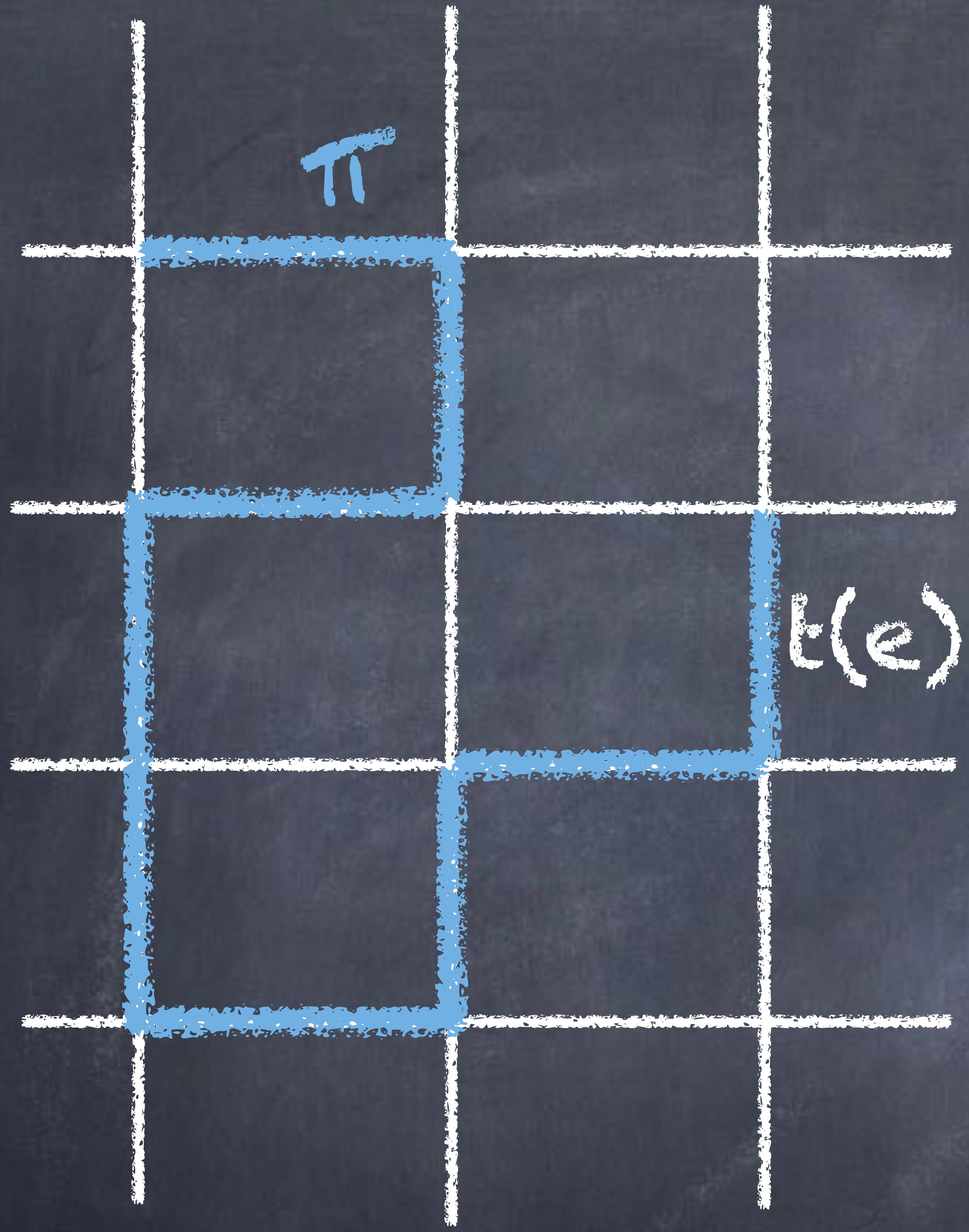
i.i.d. (enough moments, >d)



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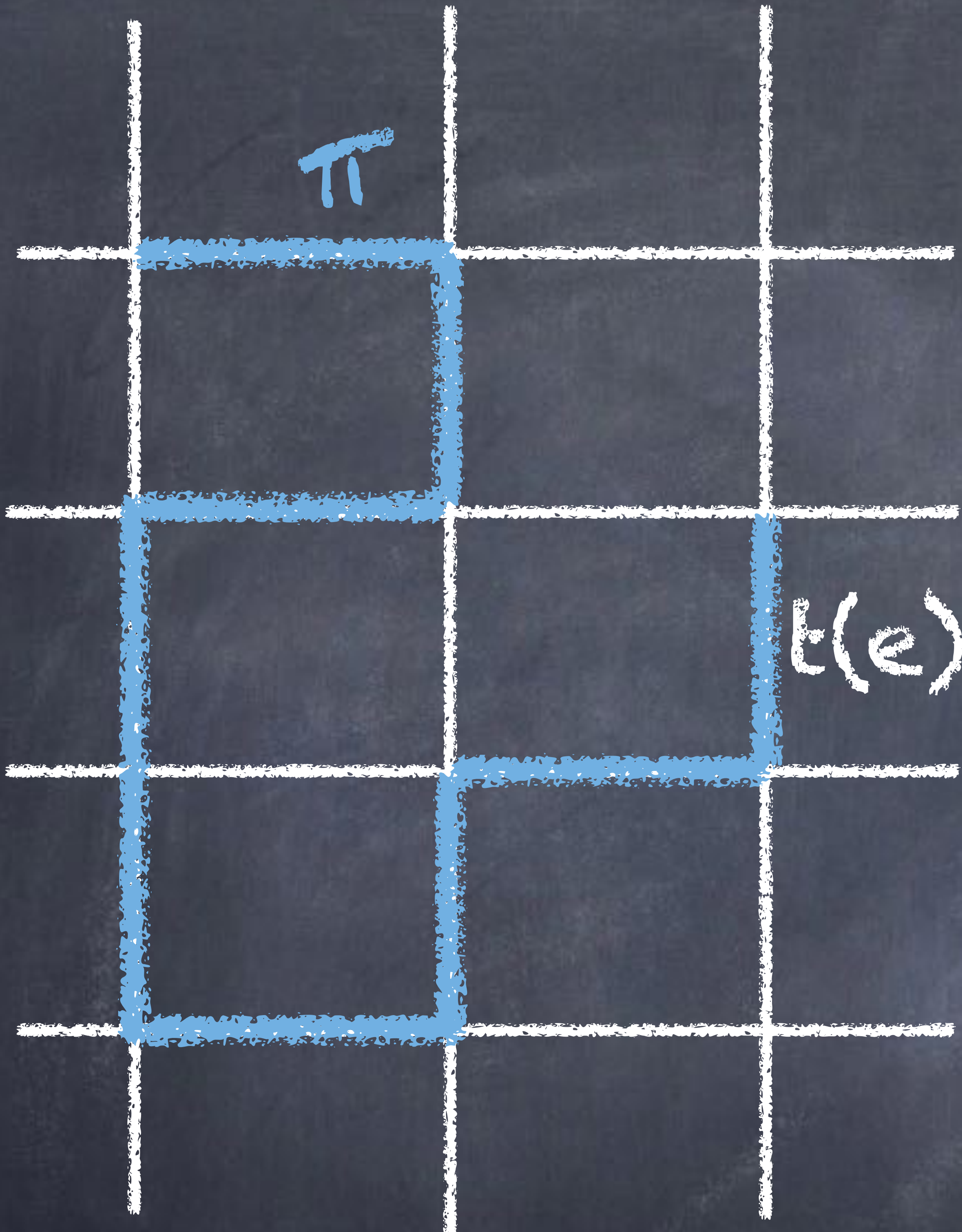

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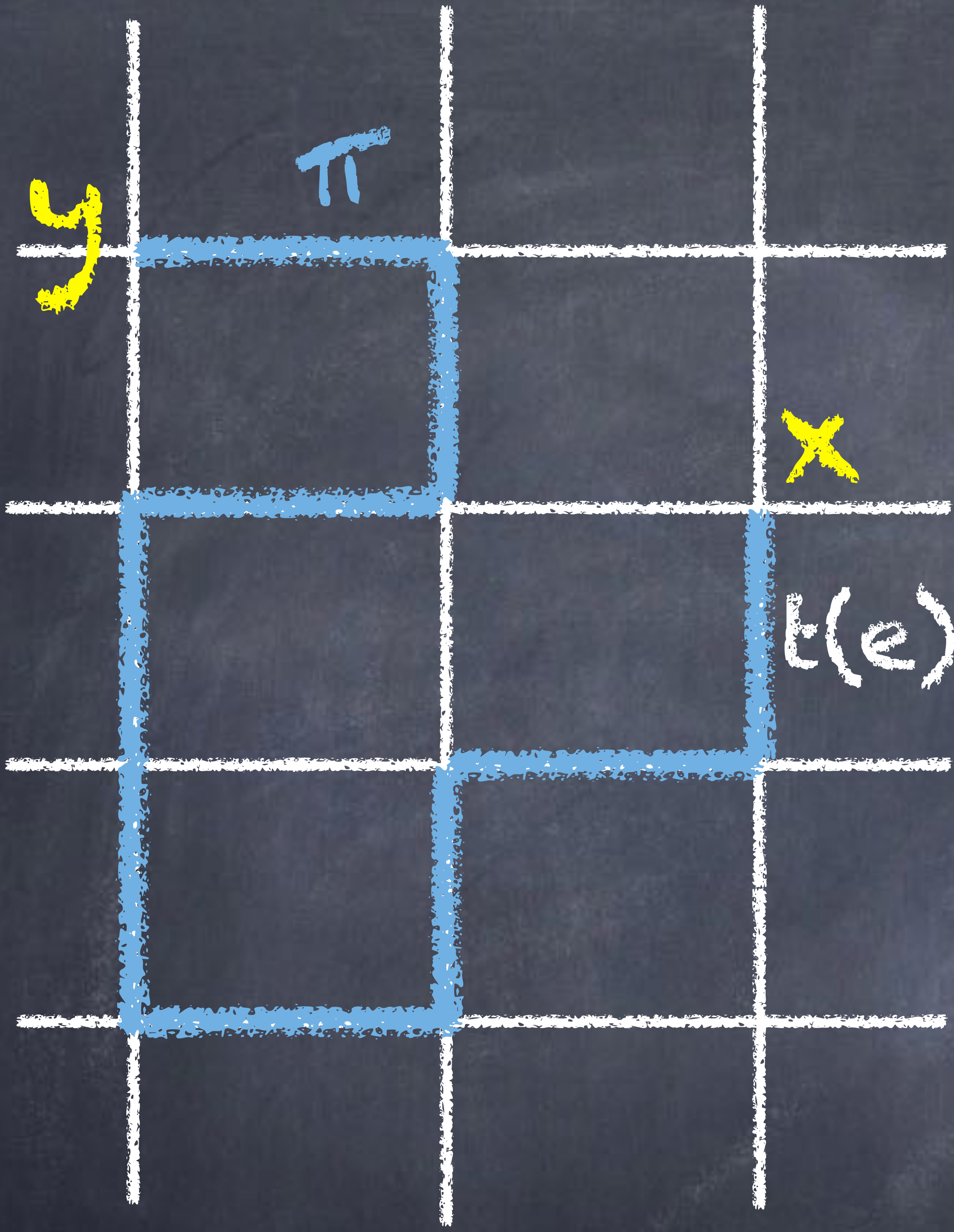
$$t(\pi) = \sum t(e)$$



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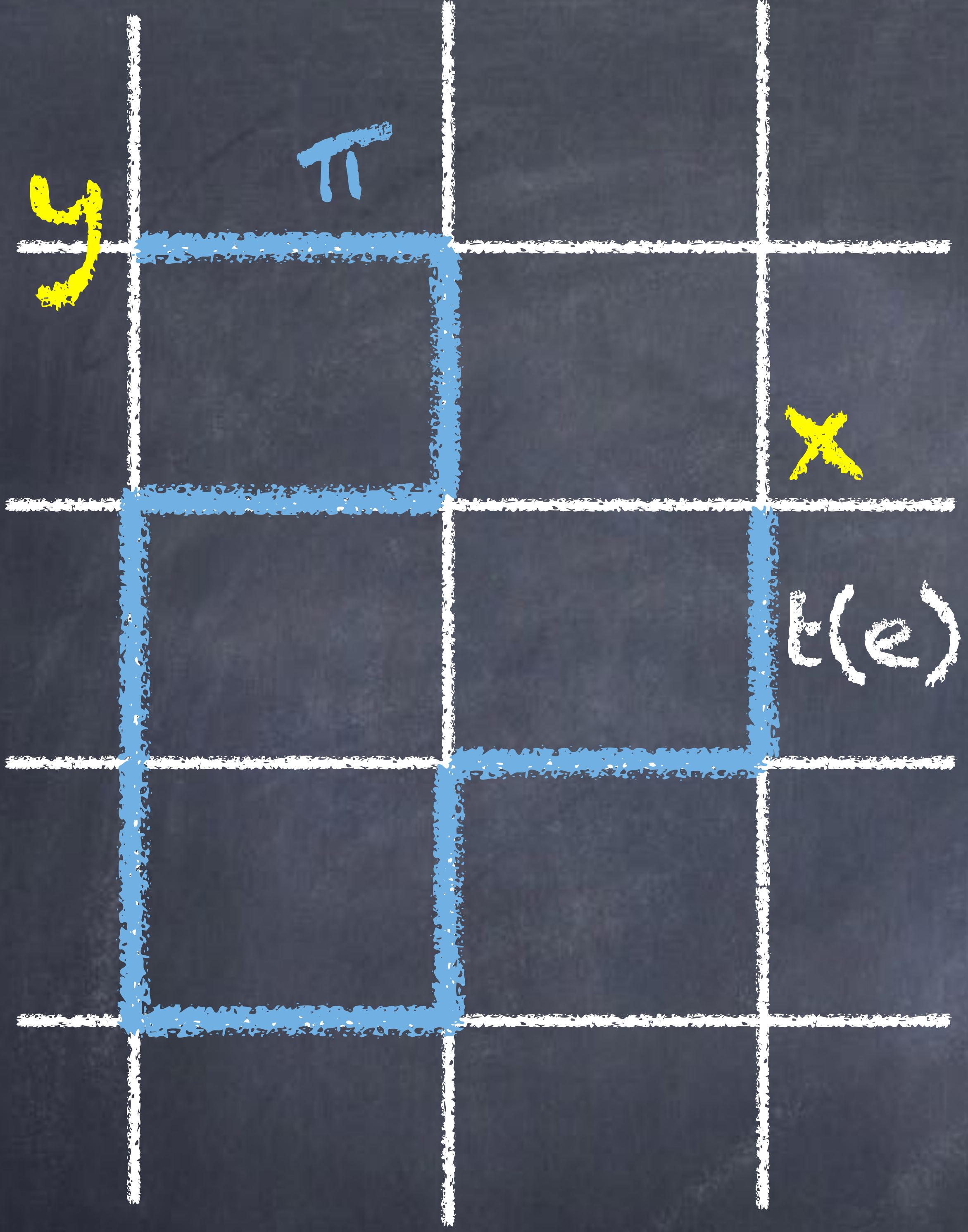

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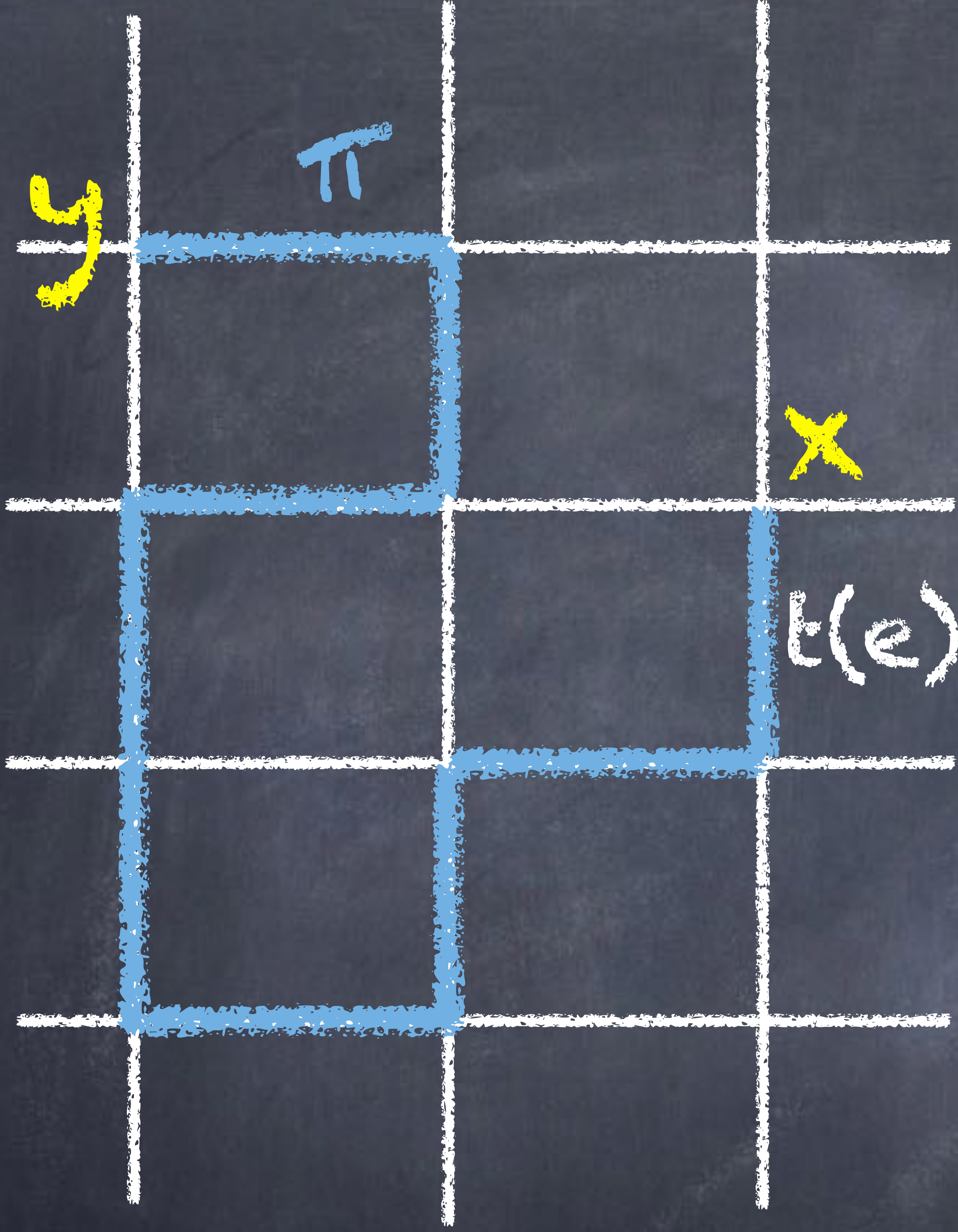

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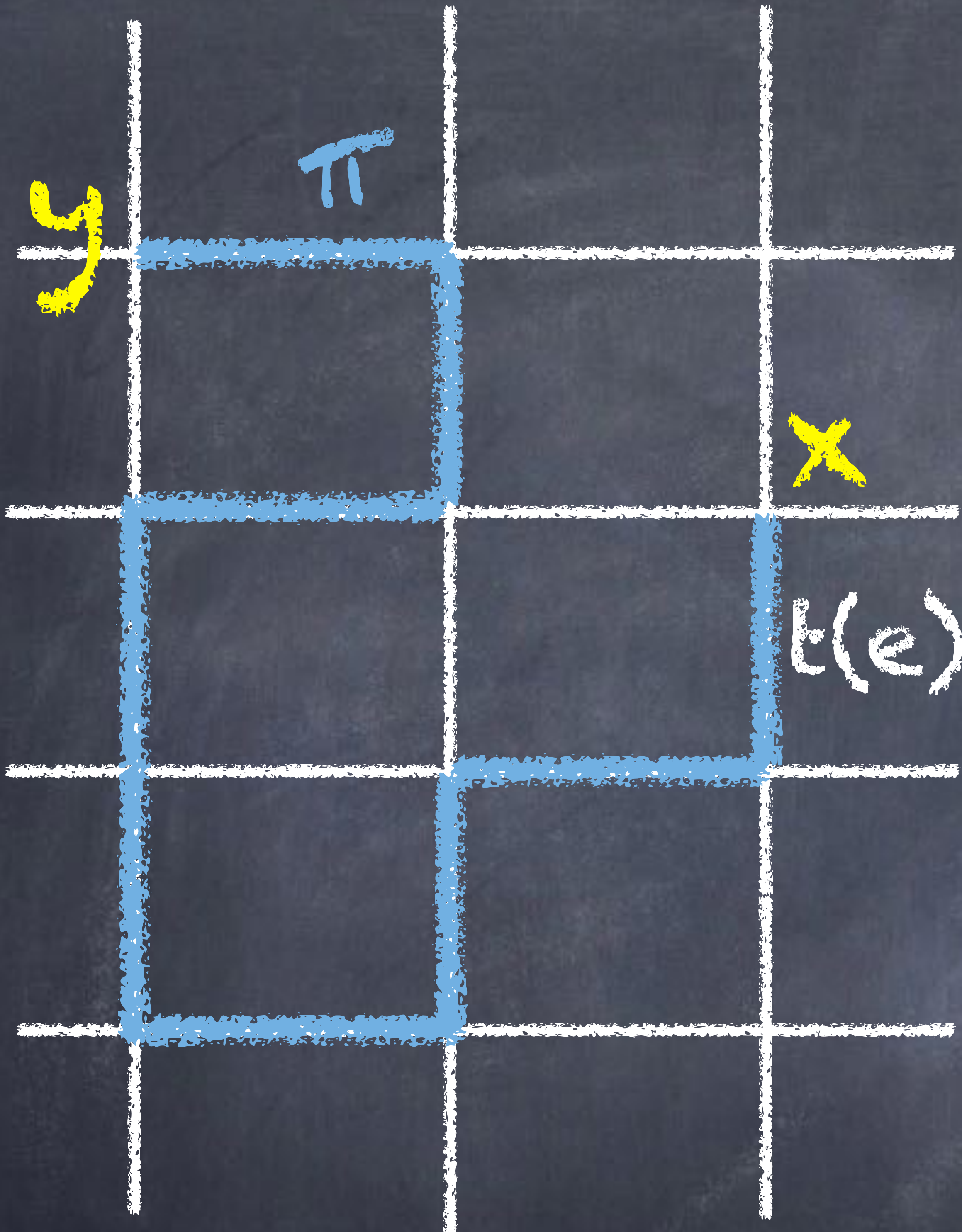

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i.i.d. (enough moments, >d)

$t(e) \geq 0$

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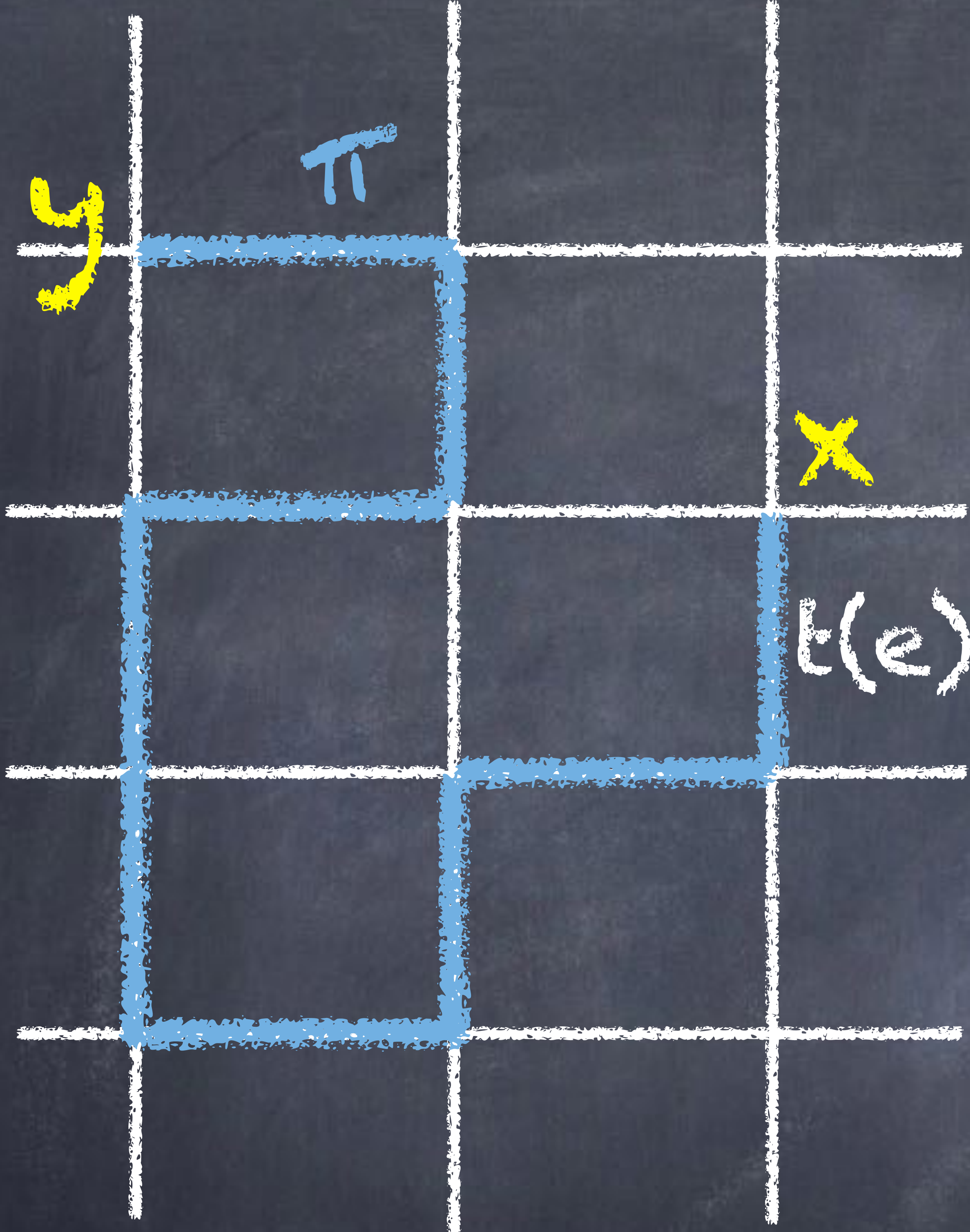
$$r_0 = \operatorname{essinf} t(e)$$



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If $r_0 = 0$: $P\{t(e) = 0\} < p_c$ (bond percolation probability)

Shape Theorem (Richardson '73, Cox-Durrett '81)

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$T_{0,x} \leq s$

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 $/s$

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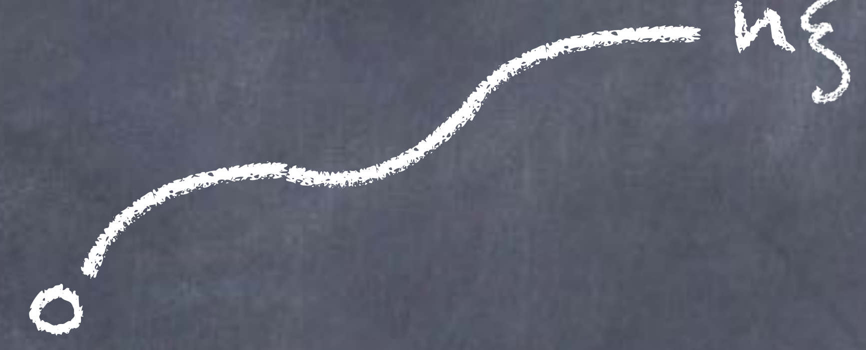
$\mu(\xi) > 0$ when $\xi \neq 0$

$$\overline{\lim_{|x|\to\infty}} \frac{|T_{0,x} - \mu(x)|}{|x|} = 0 \quad \text{a.s.}$$


nξ
o

Interested in $l(\xi) = \lim \frac{|\pi_{0,n\xi}|}{n}$ exists??



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Kesten:

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$\bar{l}(\xi) = \operatorname{esssup} \overline{\lim} \frac{\bar{L}_{0,n\xi}}{n}$

Kesten: $\bar{l}(\xi) \leq C|\xi|$ (LD bounds: $P\{\bar{L}_{0,x} \geq C|x|\} \leq e^{-c|x|}$)

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Q. $\underline{l}(\xi) = \overline{l}(\xi)$?

$$G_{x,(n),y} = \inf_{|\pi|=n} t(\pi)$$

$$G_{x,(n),y} = \inf_{|\pi|=n} t(\pi) \qquad G^{\circ}_{x,(n),y} = \inf_{|\pi|\le n} t(\pi)$$

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⚠ G and G° can repeat edges

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bridges G and T

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⚠ G and G^{o} can repeat edges

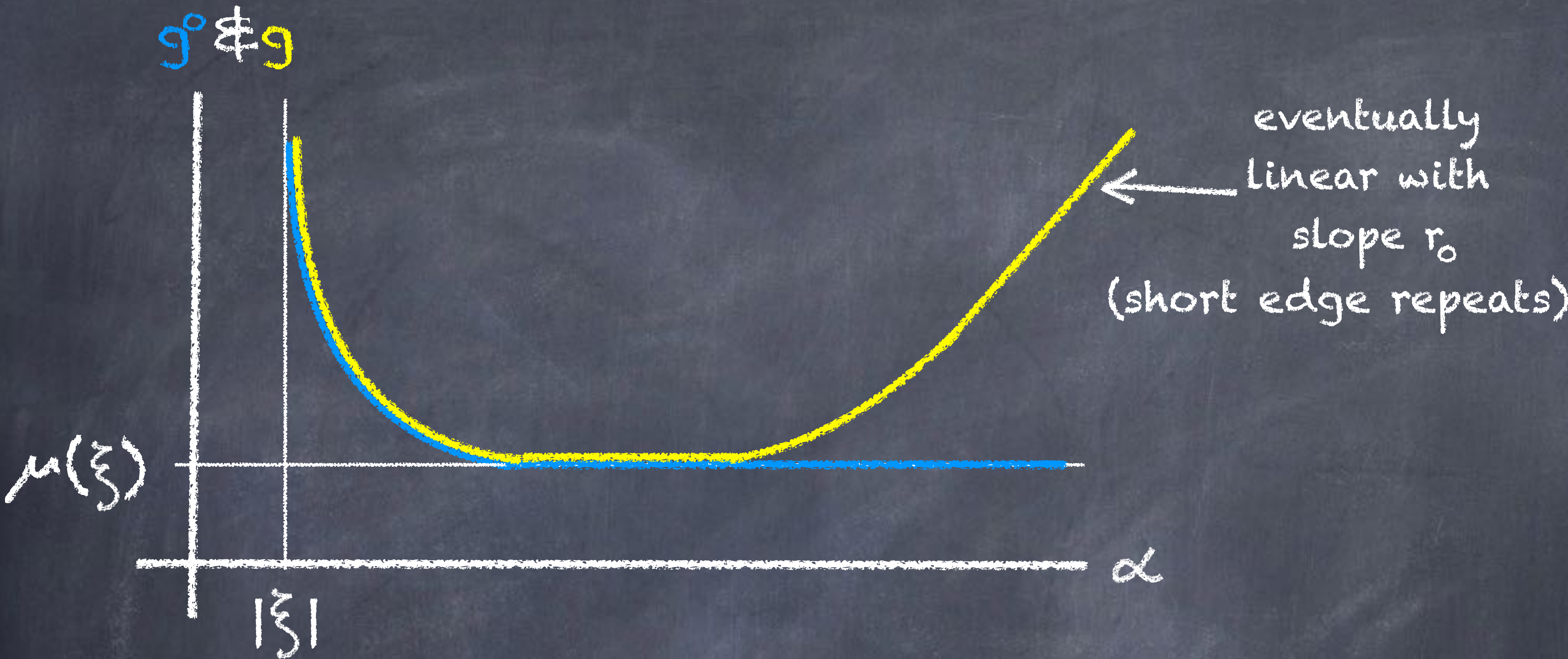
bridges G and T

$$\frac{G_{0,(n\alpha),n\xi}}{n} \longrightarrow \alpha g\left(\frac{\xi}{\alpha}\right)$$

$$\frac{G^{o}_{0,(n\alpha),n\xi}}{n} \longrightarrow \alpha g^{o}\left(\frac{\xi}{\alpha}\right)$$

g°
μ(ξ)
|ξ|
α

g° & g
μ(ξ)
α
|ξ|



g° & g
$\mu(\xi)$
$|\xi|$
α
eventually linear with slope r_0 (short edge repeats)

g° & g
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match if $r_0=0$


g° & g
$\mu(\xi)$
$|\xi|$
$\underline{\lambda}(\xi)$
$\bar{\lambda}(\xi)$
α
eventually linear with slope r_0 (short edge repeats)
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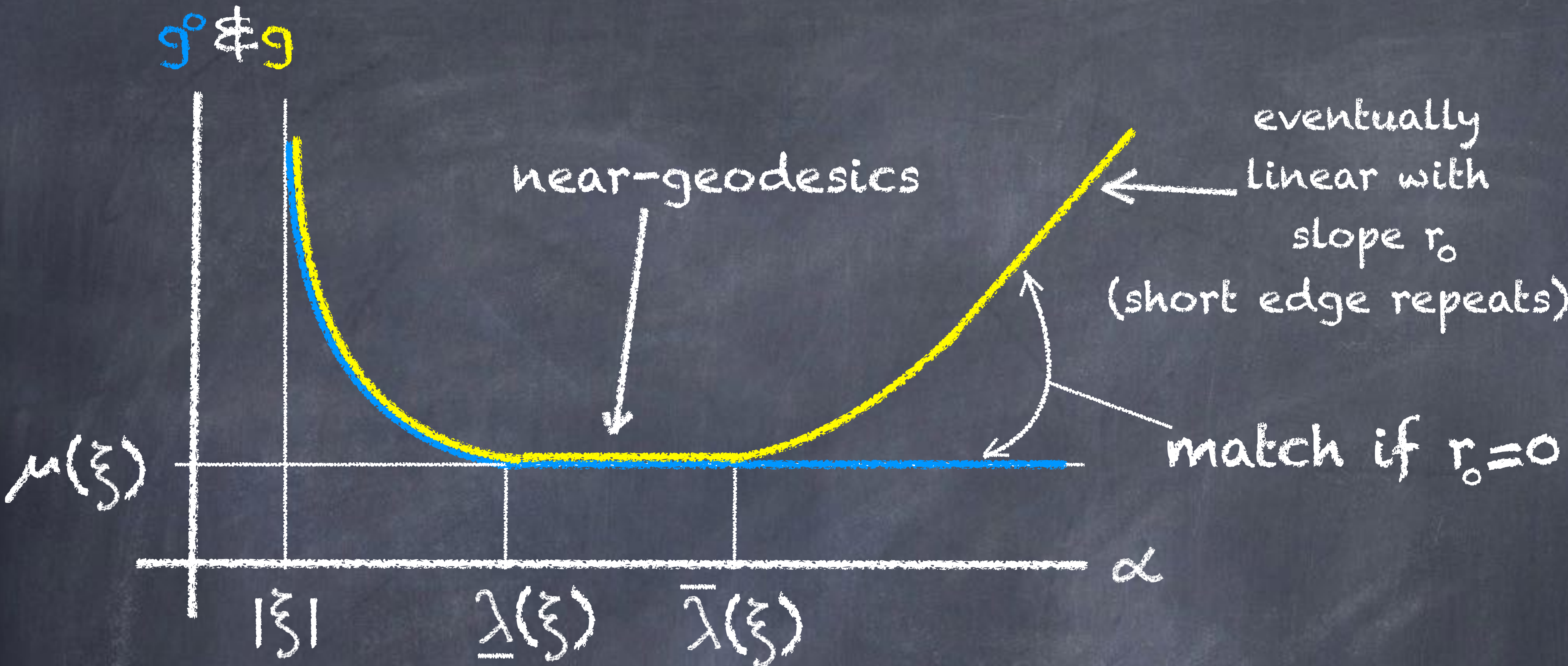
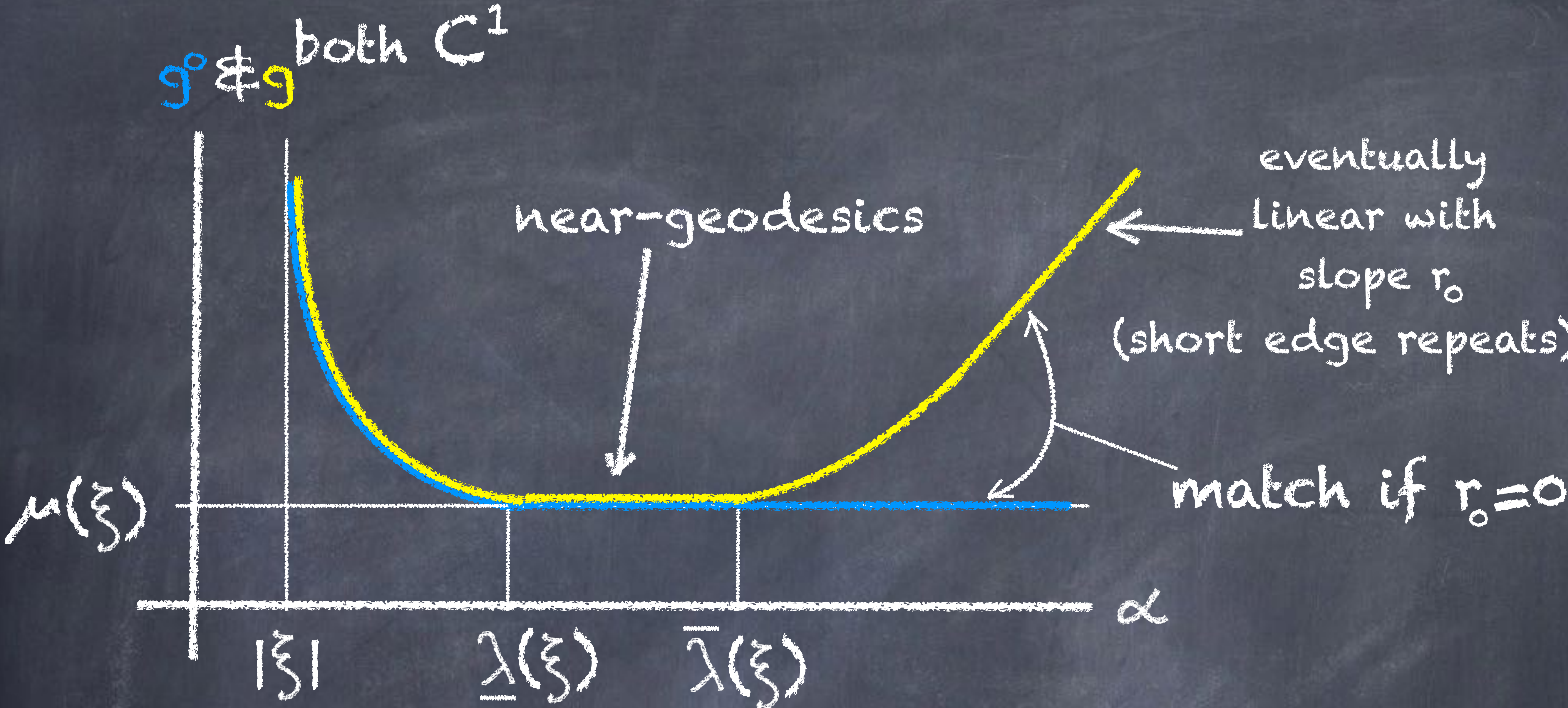
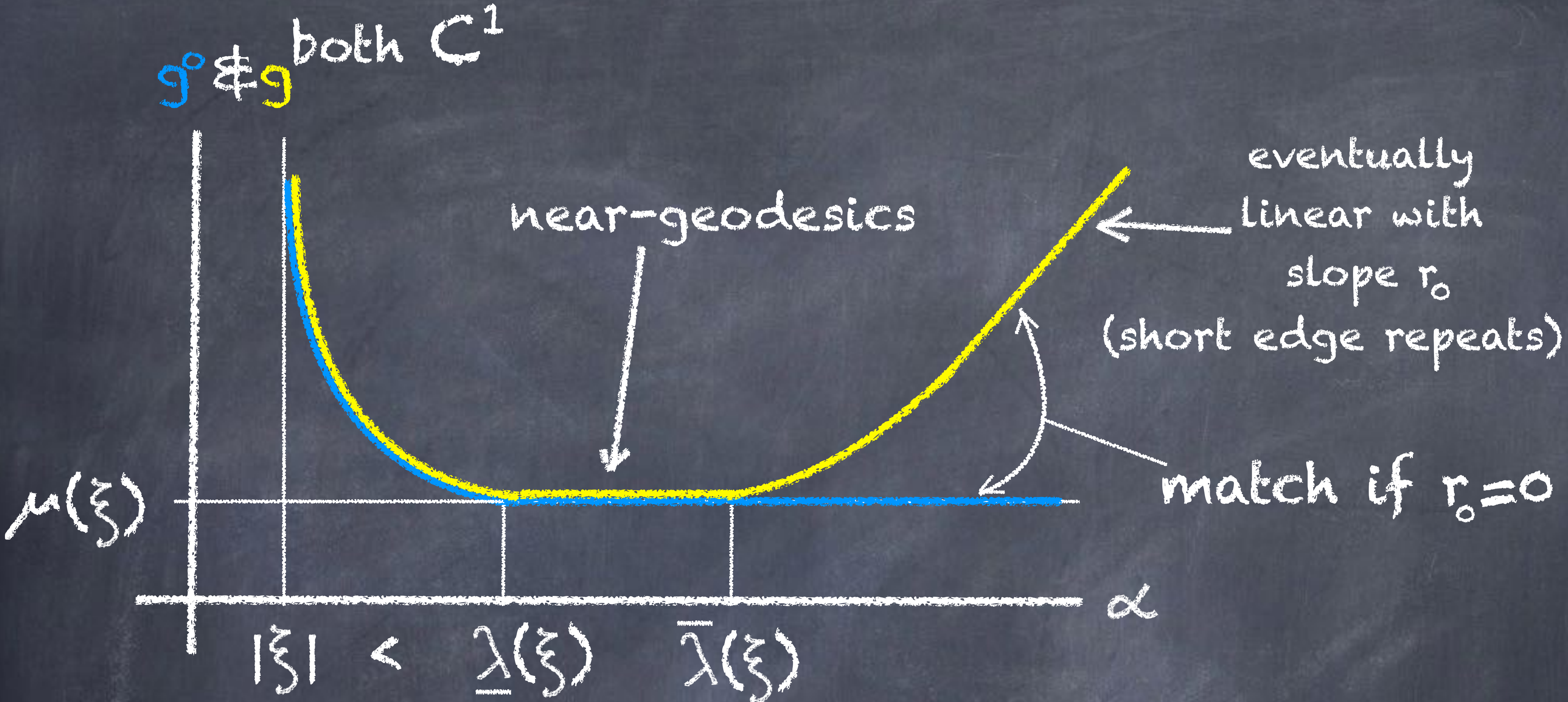
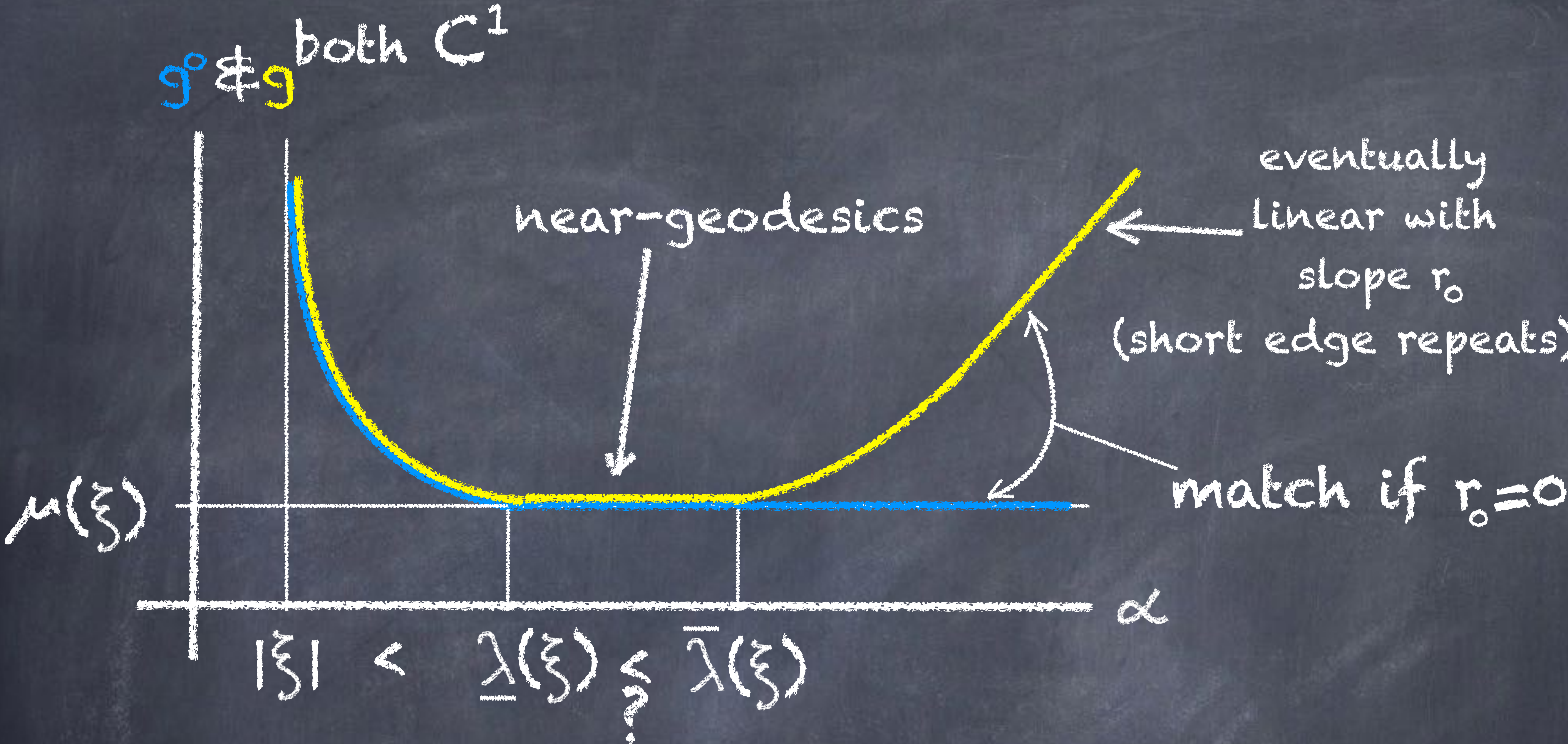

g° & g
near-geodesics
eventually linear with slope r_0
(short edge repeats)
match if $r_0=0$
$\mu(\xi)$
$|\xi|$
$\underline{\lambda}(\xi)$
$\bar{\lambda}(\xi)$
α



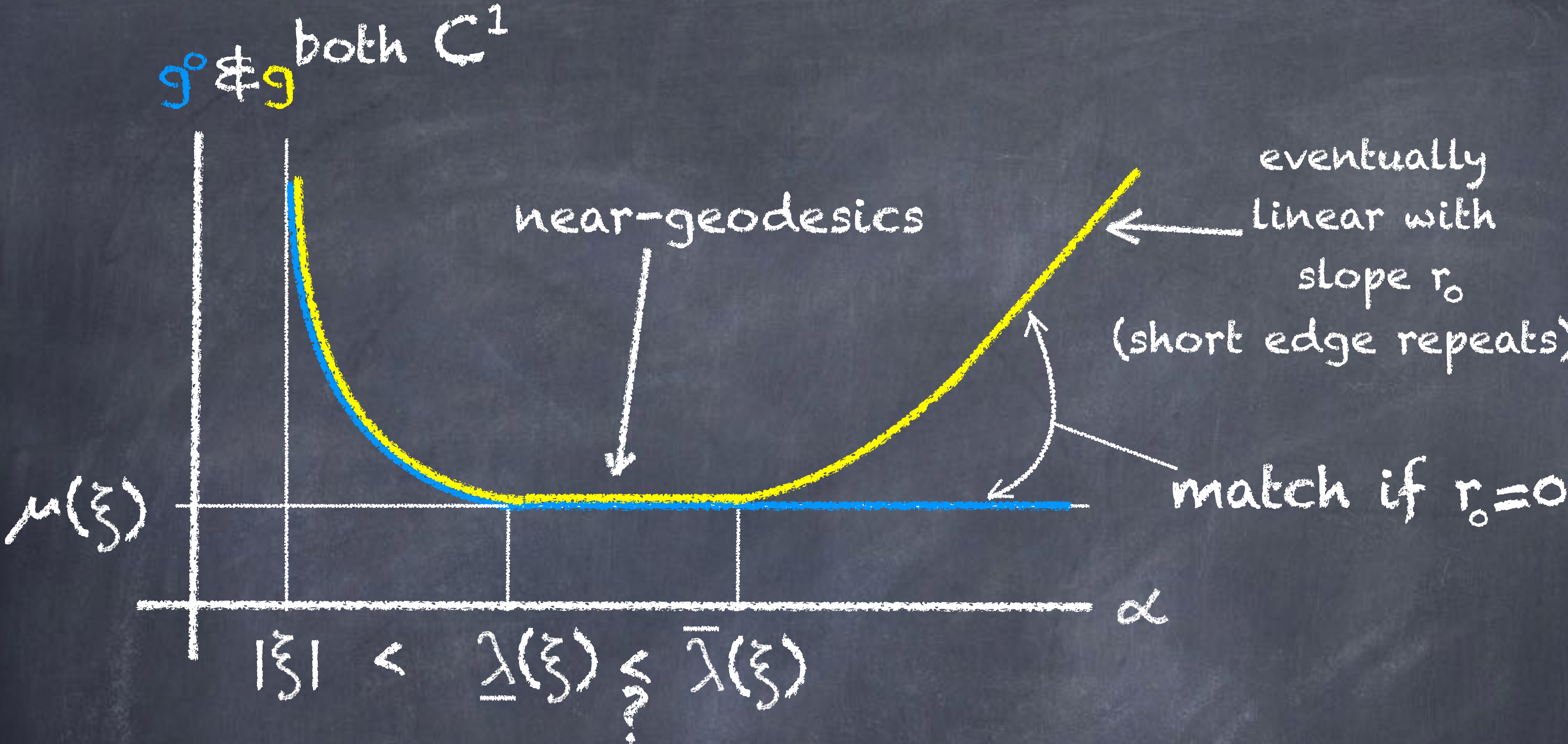
g^0 & g both C^1
near-geodesics
eventually linear with slope r_0 (short edge repeats)
match if $r_0=0$
$\mu(\xi)$
$|\xi|$
$\underline{\lambda}(\xi)$
$\overline{\lambda}(\xi)$
α



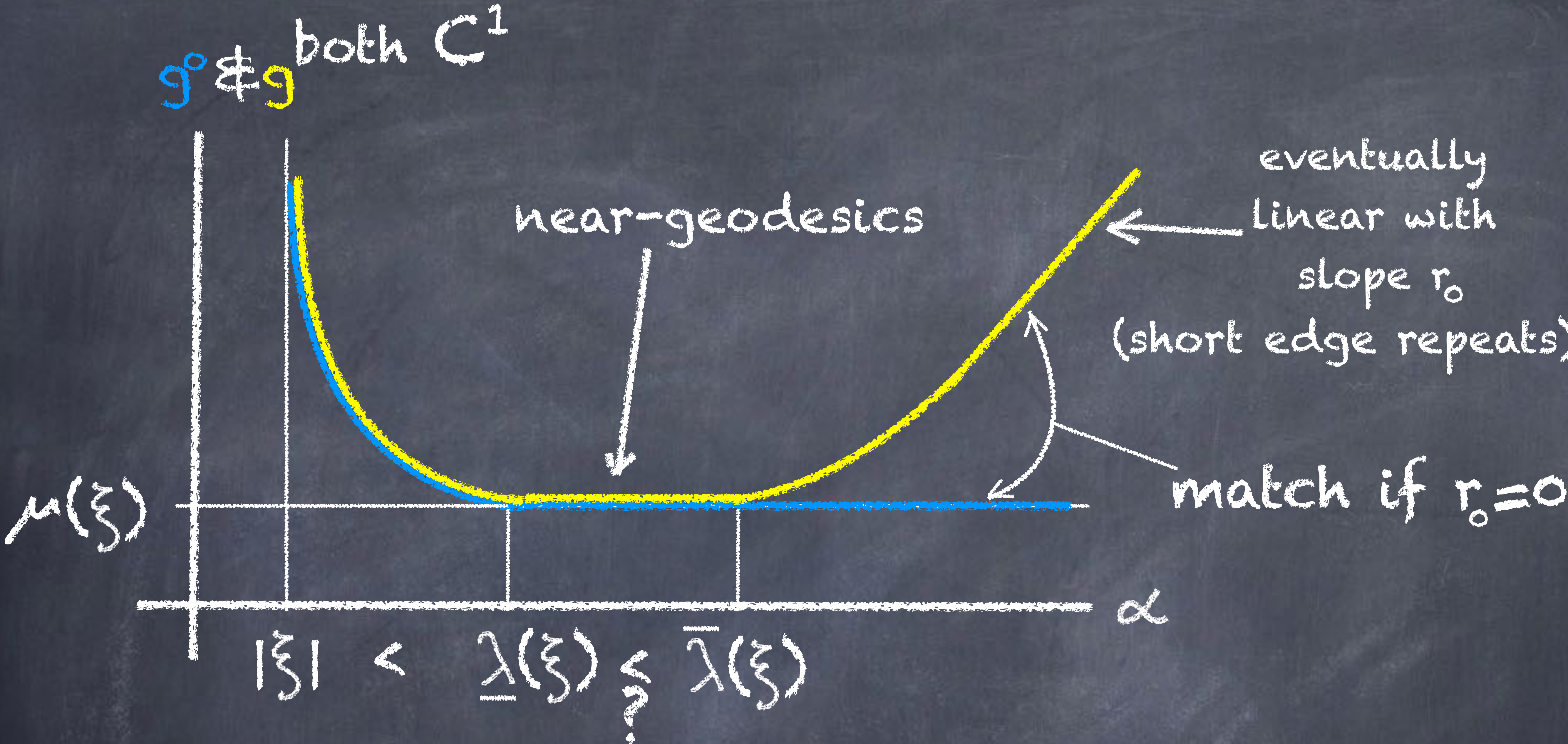
g^0 & g both C^1
near-geodesics
eventually linear with slope r_0
(short edge repeats)
match if $r_0=0$
$\mu(\xi)$
α
$|\xi| < \underline{\lambda}(\xi)$
$\bar{\lambda}(\xi)$



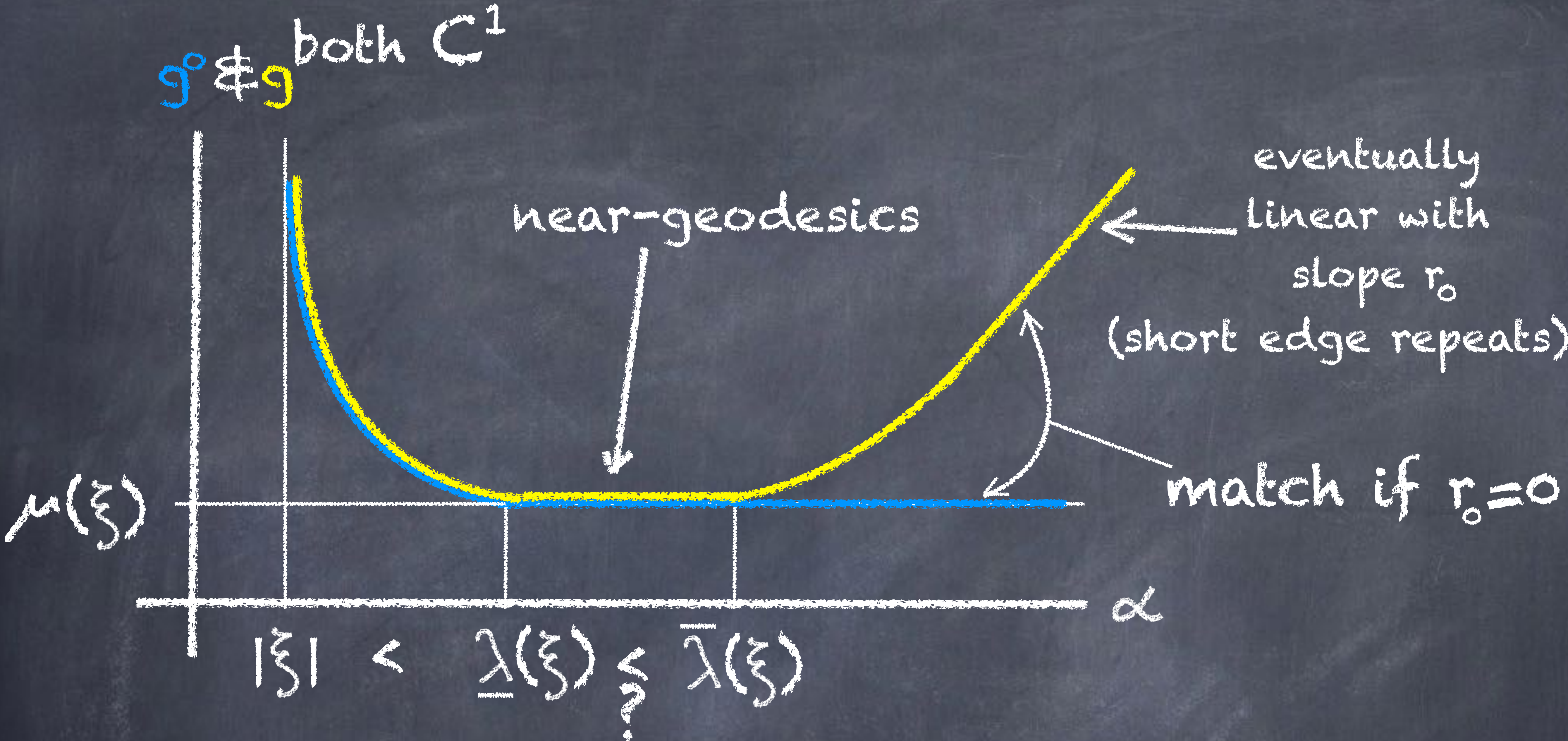
g° & g both C¹
near-geodesics
eventually linear with slope r₀ (short edge repeats)
match if r₀=0
μ(ξ)
α
|ξ| < λ(ξ) ≤? λ̄(ξ)



$\leq \underline{l}(\xi) \leq \overline{l}(\xi) \leq$

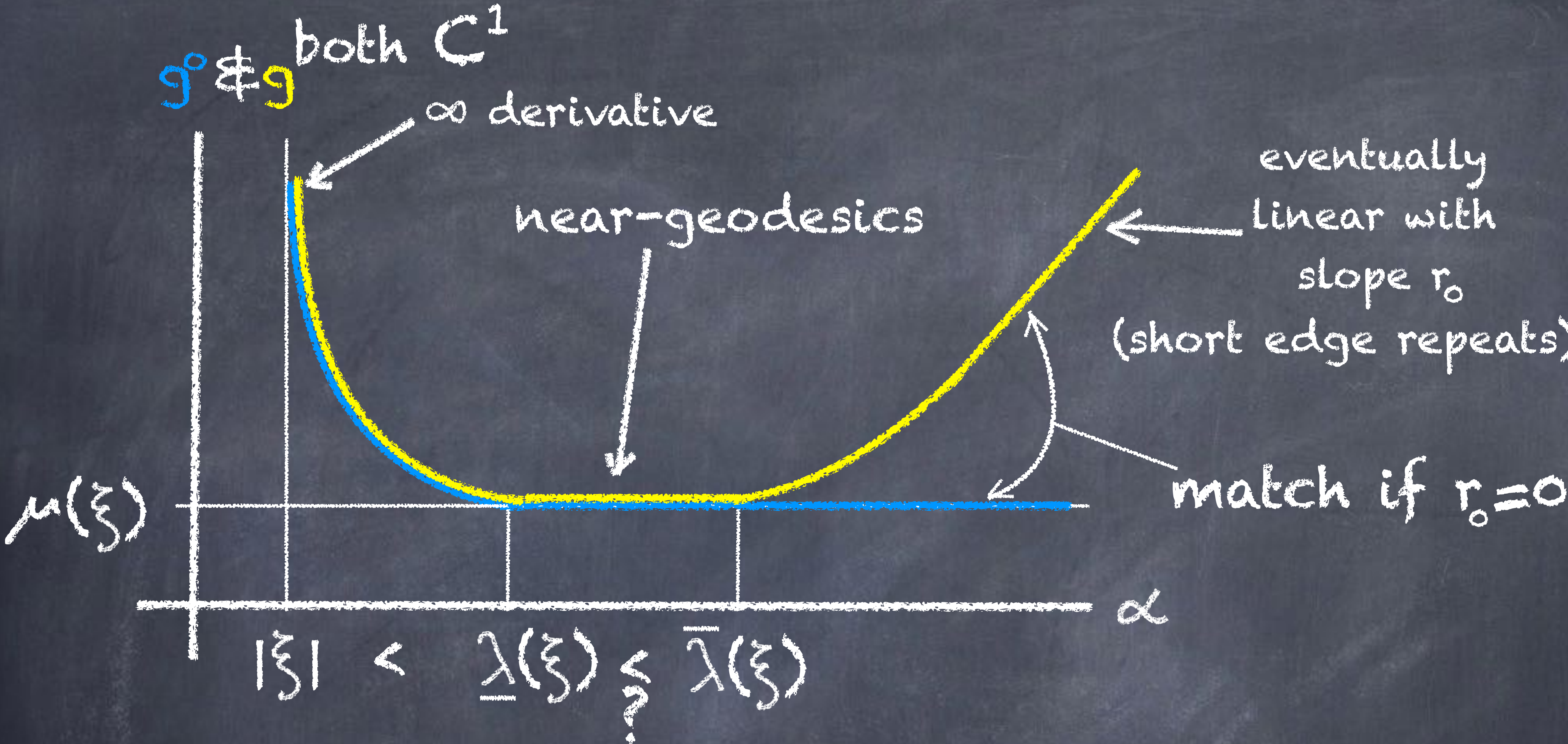


$$\underline{\lambda}(\xi) \leq \underline{l}(\xi) \leq \overline{l}(\xi) \leq \overline{\lambda}(\xi)$$



$$|\xi| < \underline{\lambda}(\xi) \leq \underline{l}(\xi) \leq \overline{l}(\xi) \leq \overline{\lambda}(\xi) < \infty$$

↑ iff $r_0 > 0$



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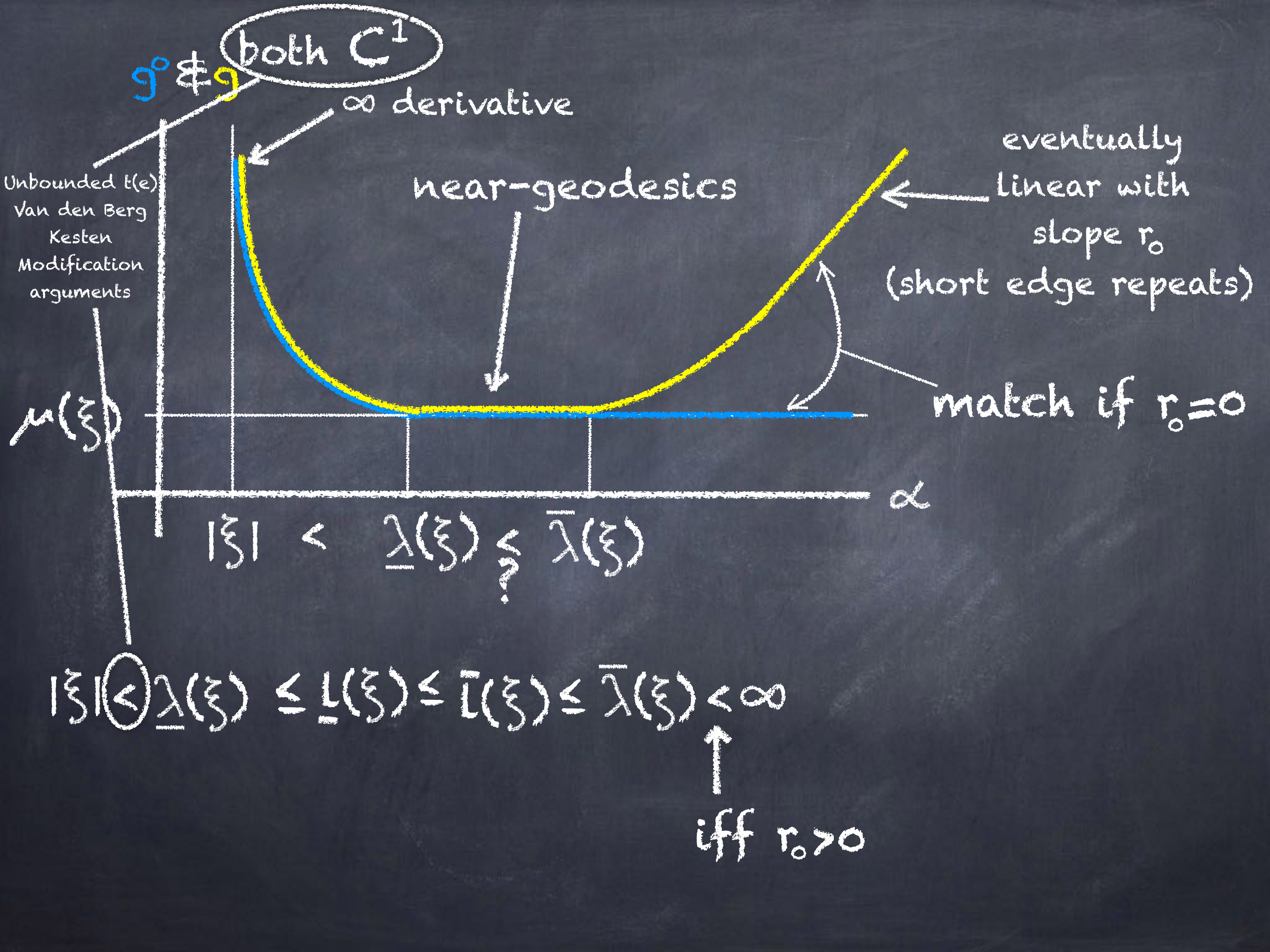
both C^1
g° & g
∞ derivative
Unbounded t(e)
Van den Berg
Kesten
Modification
arguments
near-geodesics
eventually
linear with
slope r_0
(short edge repeats)
$\mu(\xi)$
match if $r_0=0$
α
$|\xi| < \underline{\lambda}(\xi) \overset{?}{\leq} \overline{\lambda}(\xi)$
$|\xi| < \underline{\lambda}(\xi) \leq \underline{l}(\xi) \leq \overline{l}(\xi) \leq \overline{\lambda}(\xi) < \infty$
iff $r_0>0$

Shifting $t \longrightarrow t+b = t^{(b)} \implies$

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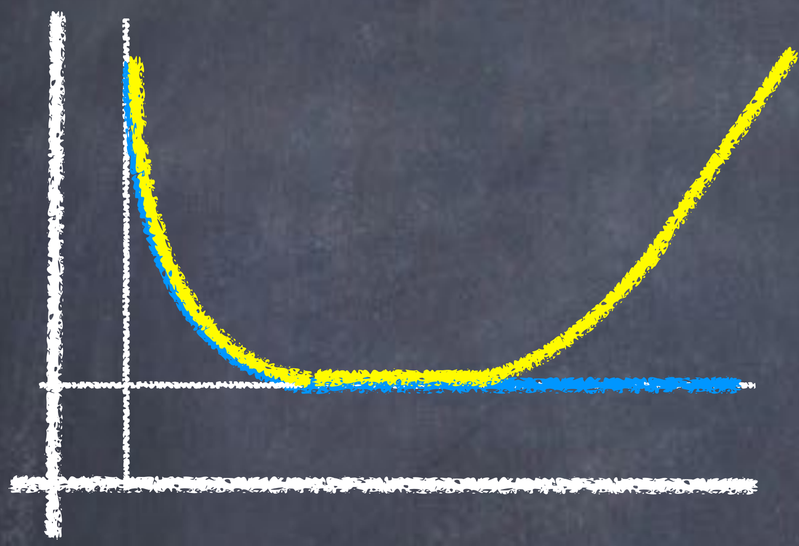

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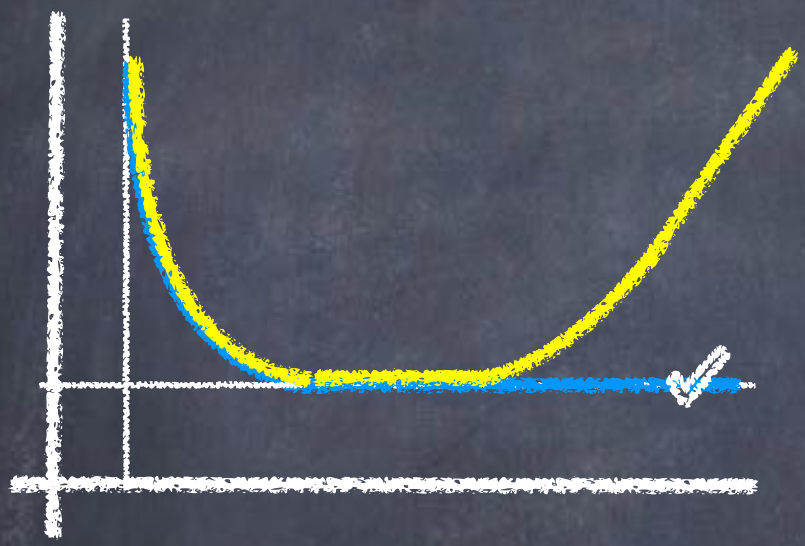

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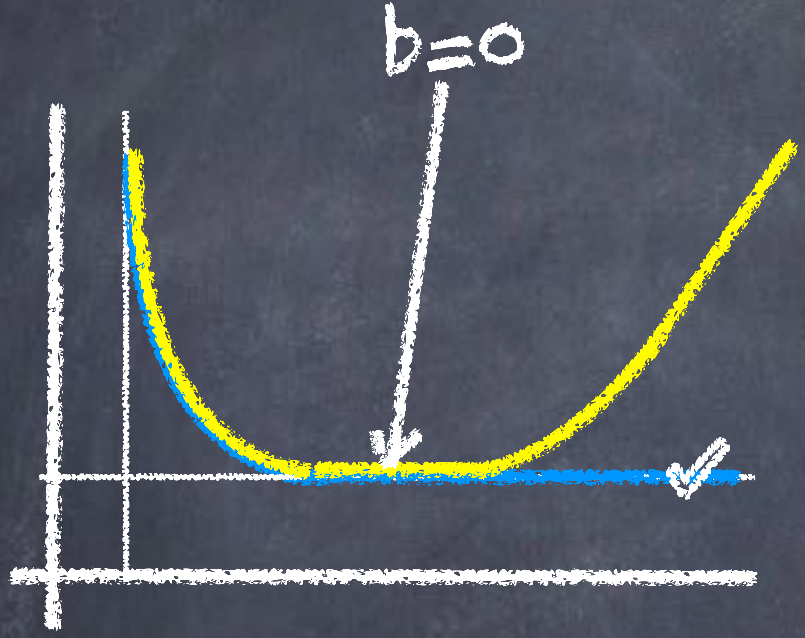
near-geodesics



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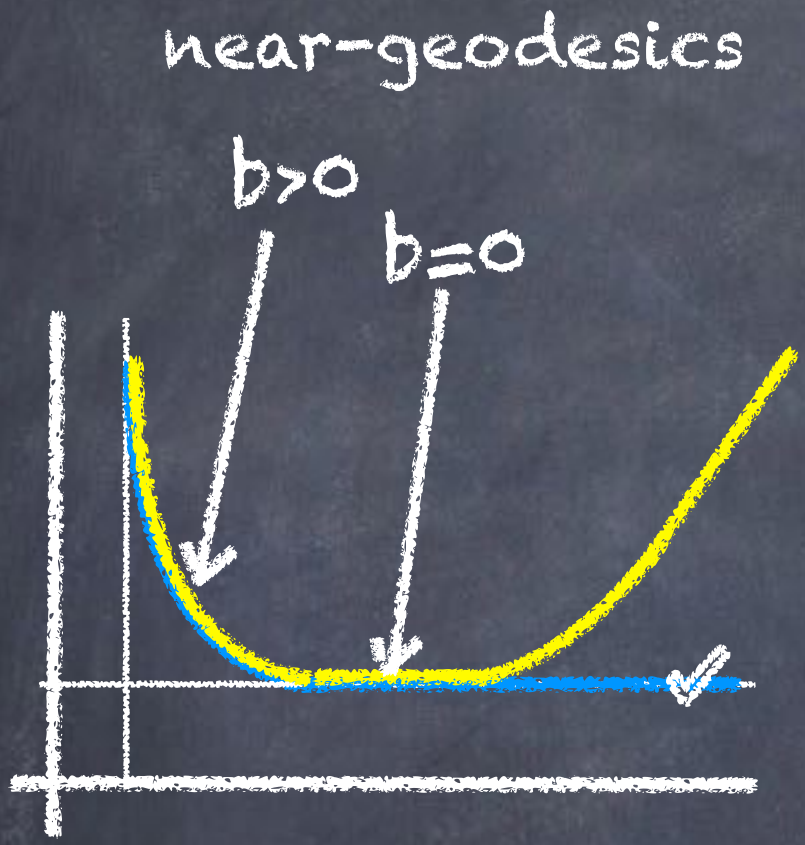

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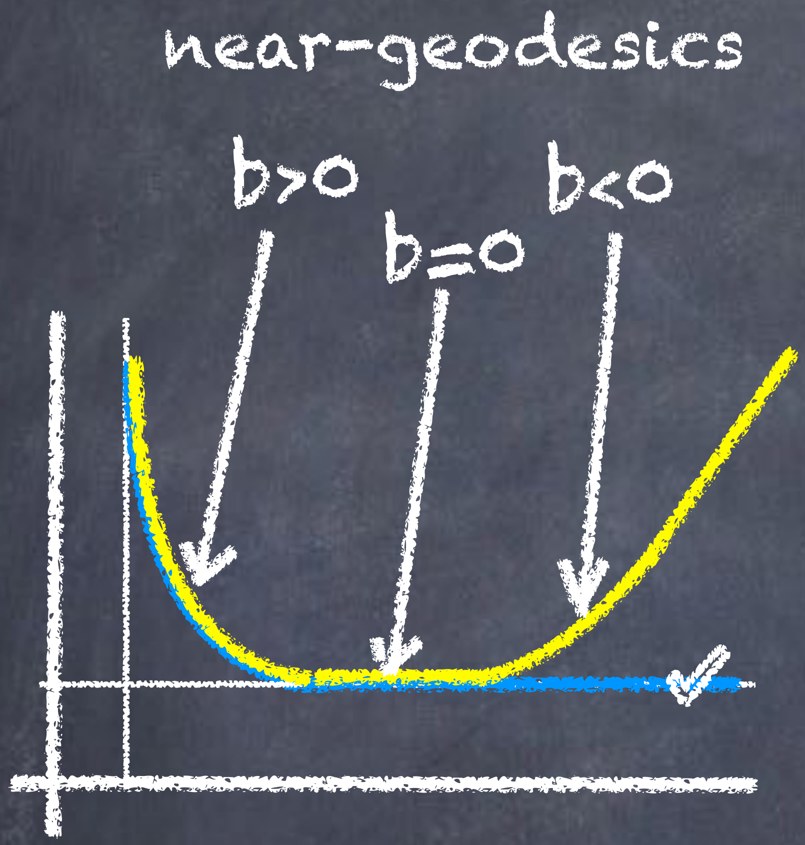

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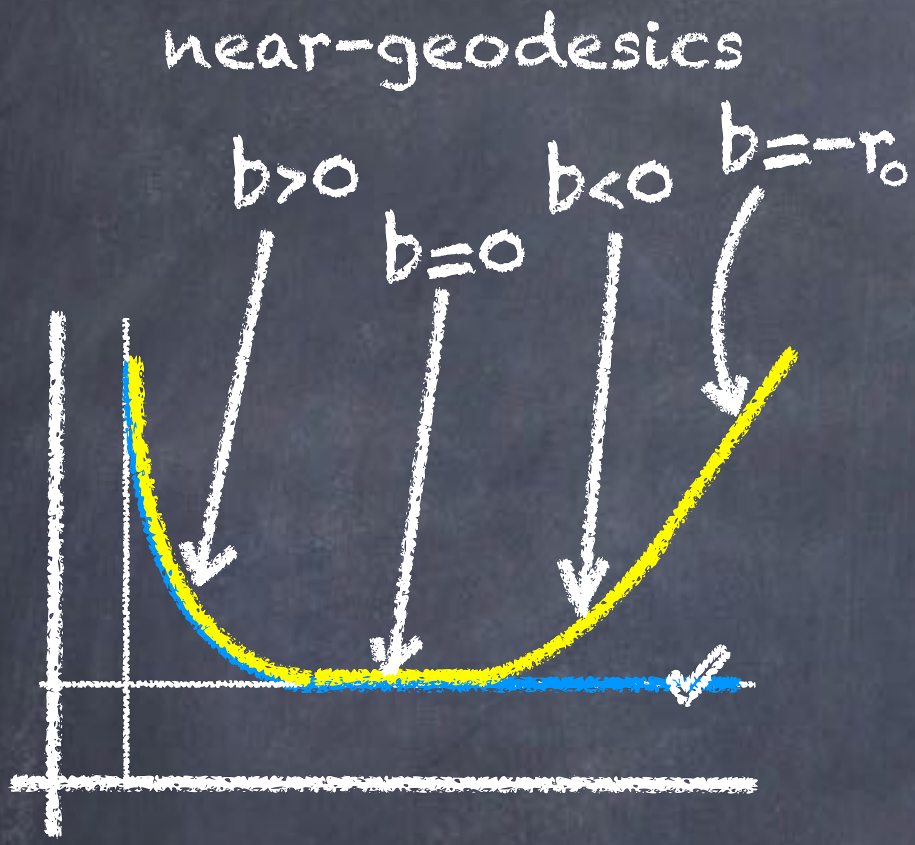
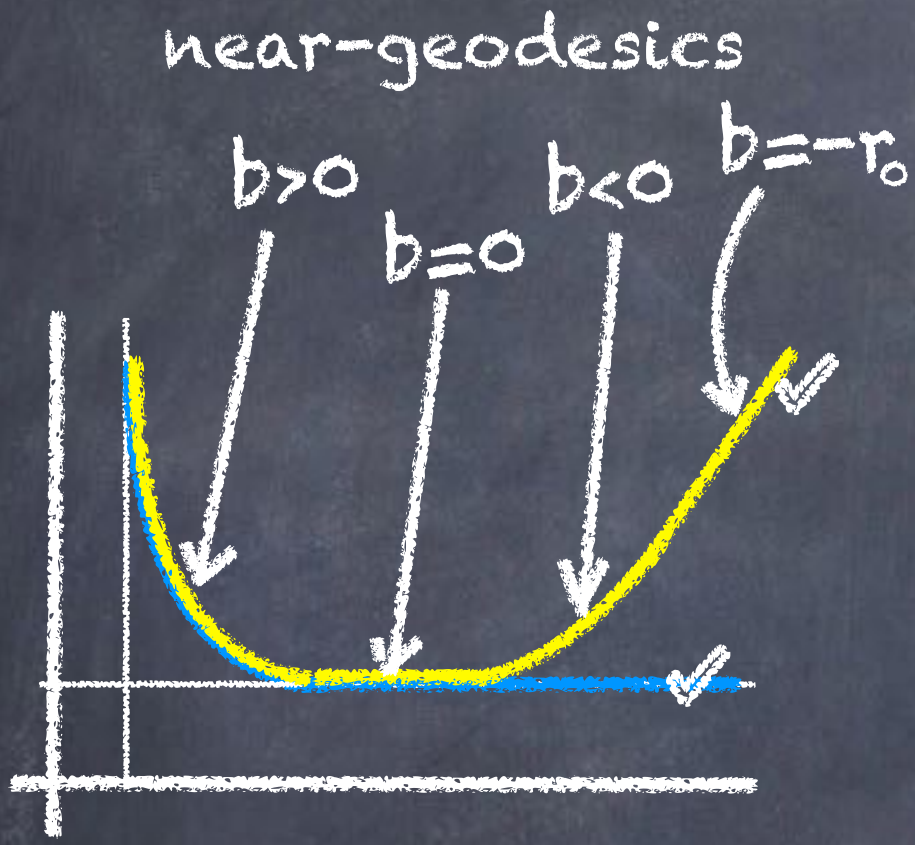


Shifting $t \longrightarrow t+b = t^{(b)} \implies \propto g(\frac{\xi}{\alpha}) \longrightarrow \propto g(\frac{\xi}{\alpha}) + b\alpha$



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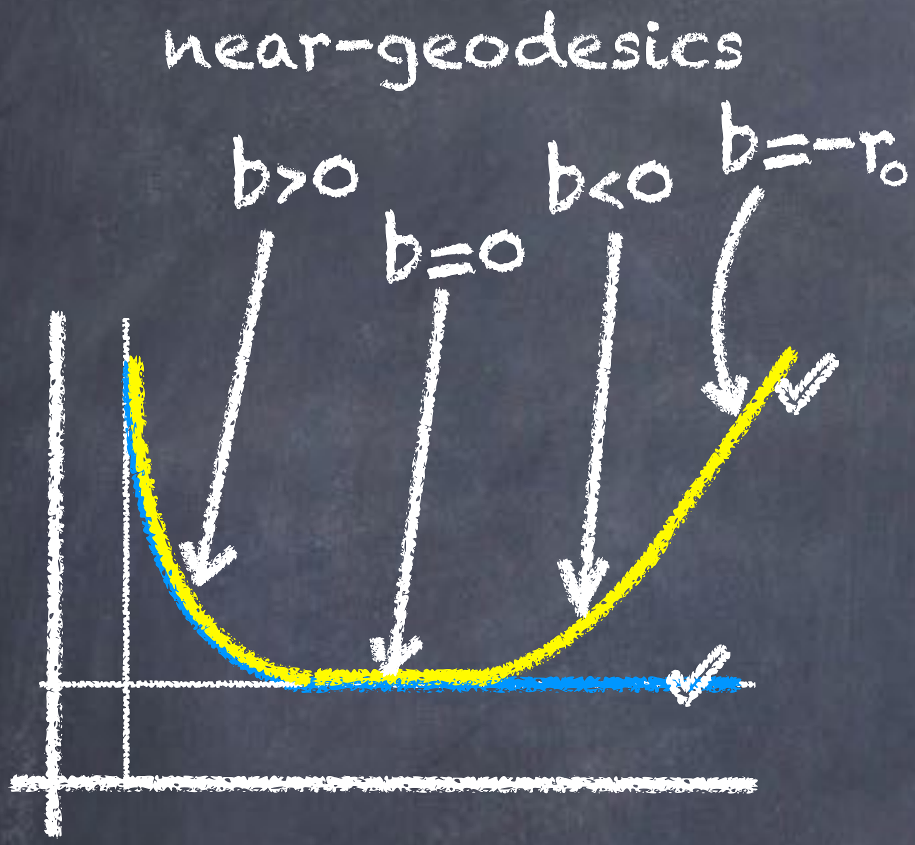

assume $r_0 = 0$

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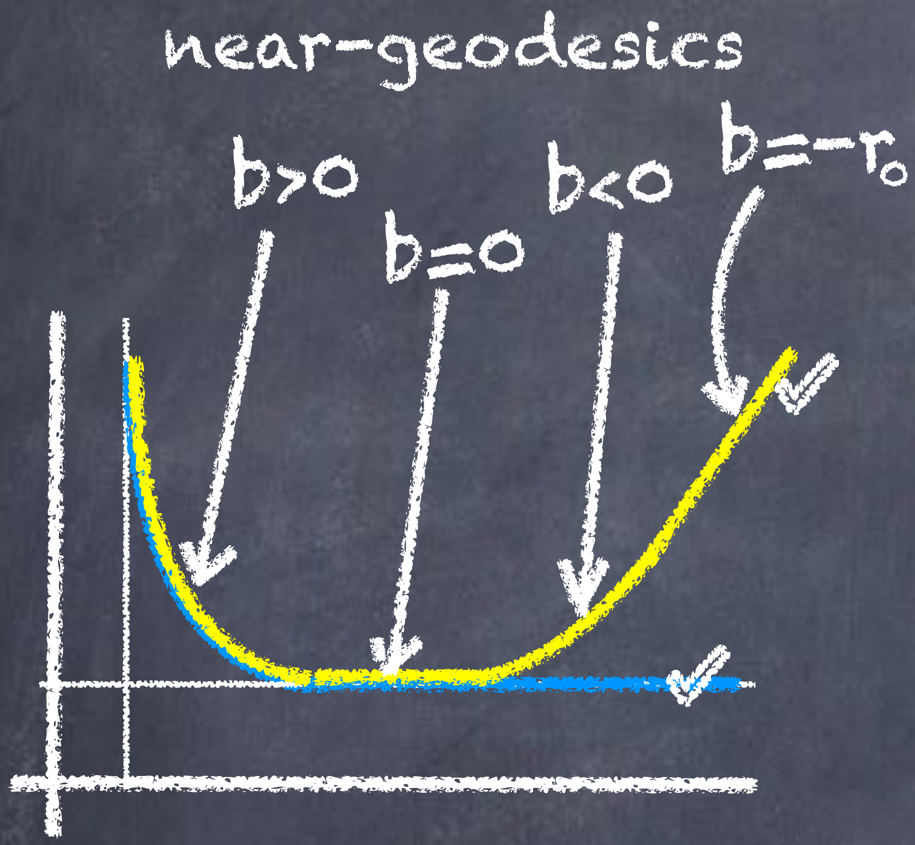

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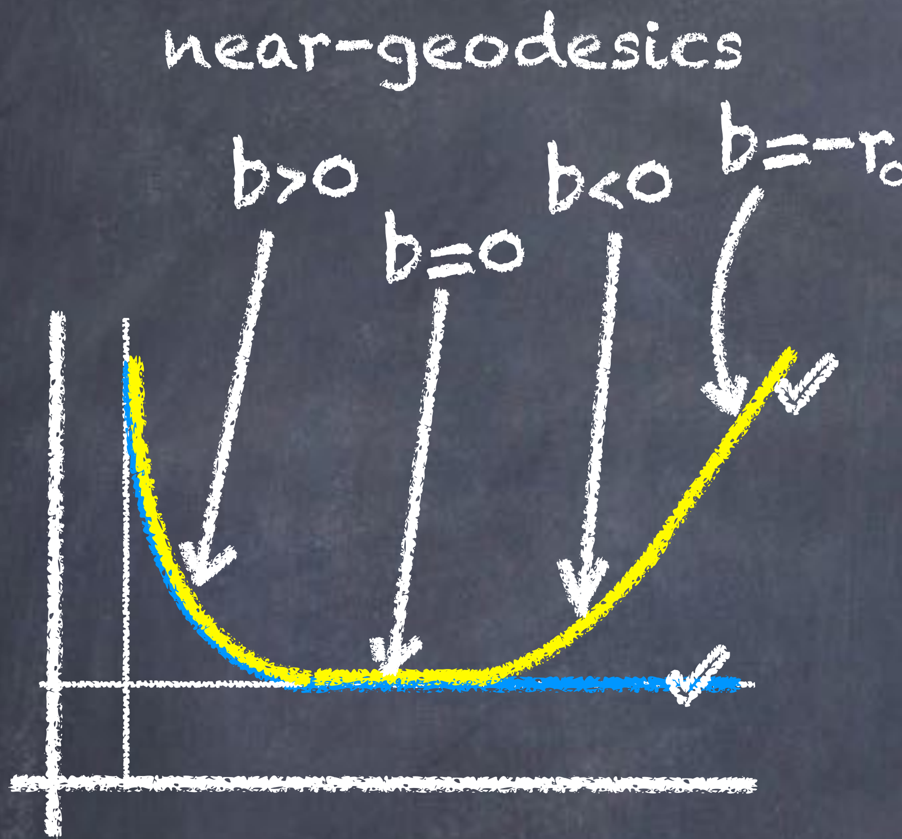
$$|\xi| < \underline{\lambda}^{(b)}(\xi) \leq \overline{\lambda}^{(b)}(\xi) \leq \underline{\lambda}^{(a)}(\xi) \leq \overline{\lambda}^{(a)}(\xi) \leq \underline{\lambda}(\xi) \leq C|\xi| \quad (b>a)$$

Shifting $t \longrightarrow t+b=t^{(b)} \implies \alpha g(\frac{\xi}{\alpha}) \longrightarrow \alpha g(\frac{\xi}{\alpha})+b\alpha$



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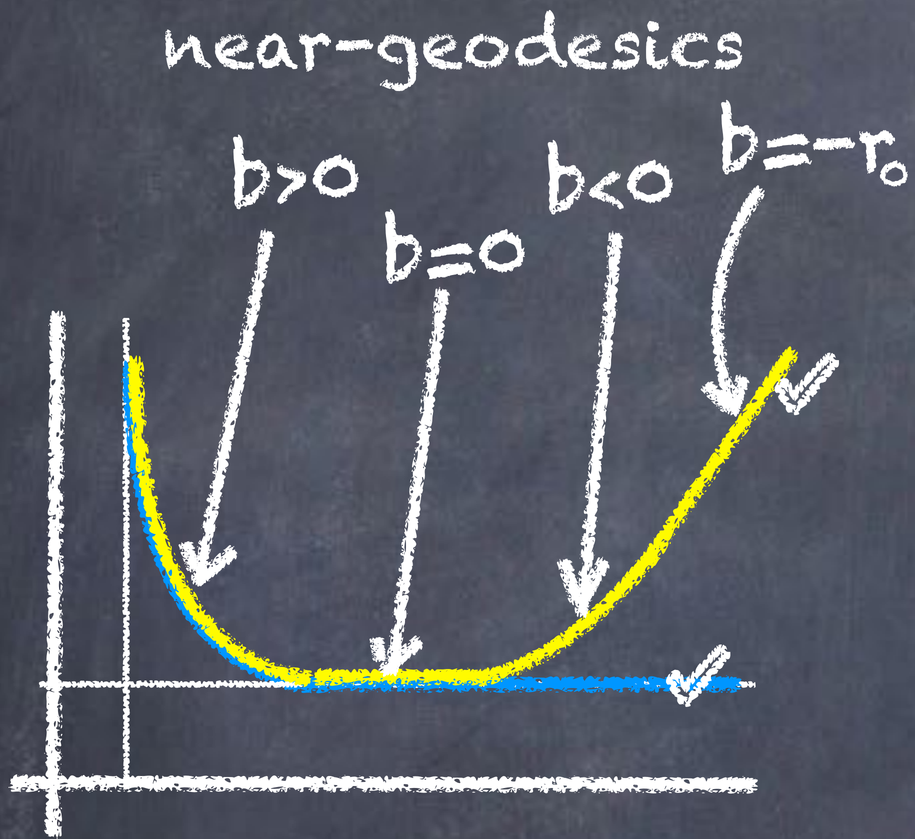
Different possible geo lengths for different shifts DO NOT MIX, even for distinct typical ω!

Shifting $t \longrightarrow t+b=t^{(b)} \Longrightarrow \alpha g(\frac{\xi}{\alpha}) \longrightarrow \alpha g(\frac{\xi}{\alpha})+b\alpha$



$$\overline{\lambda}^{(b)}(\xi) \xrightarrow[b\to\infty]{} |\xi|$$

(high weights $\Longrightarrow$ high sensitivity)

assume $r_0=0$

$$|\xi| < \underline{\lambda}^{(b)}(\xi) \le \overline{\lambda}^{(b)}(\xi) \le \underline{\lambda}^{(a)}(\xi) \le \overline{\lambda}^{(a)}(\xi) \le \underline{\lambda}(\xi) \le C|\xi| \quad (b>a)$$

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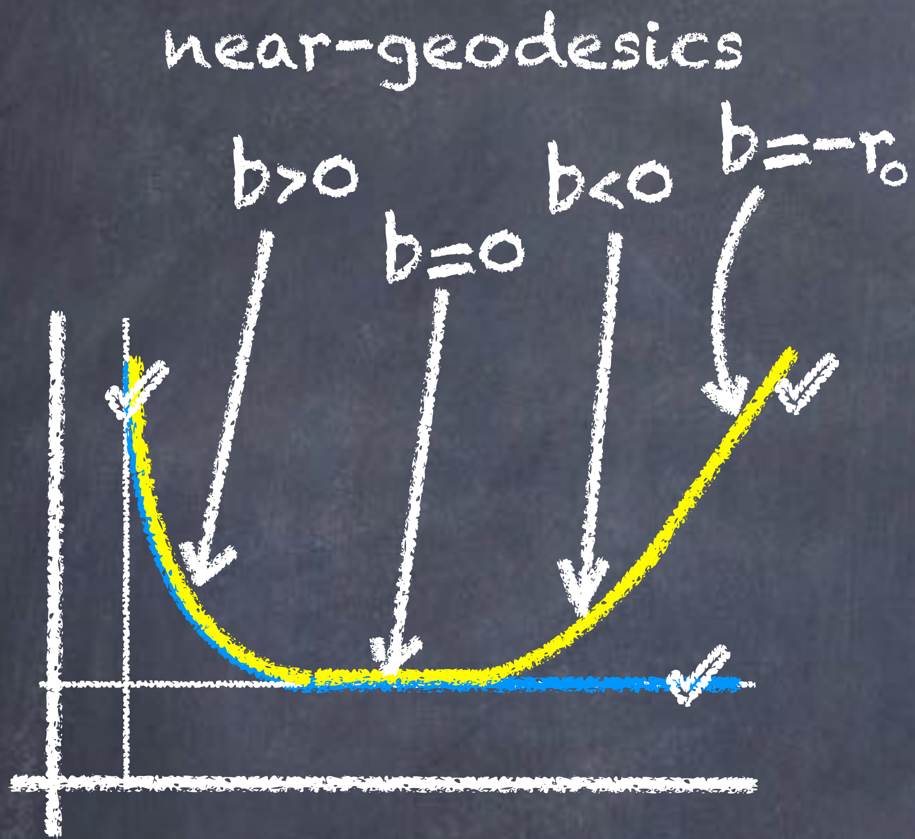

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Different possible geo lengths for different shifts DO NOT MIX, even for distinct typical ω!

$$\mu(\xi) \longrightarrow \mu^{(b)}(\xi) = \phi(b) \approx \mu(\xi) + b\text{``}L(\xi)\text{''} \qquad b \approx 0$$

$\mu(\xi) \longrightarrow \mu^{(b)}(\xi) = \phi(b) \approx \mu(\xi) + b\text{"}l(\xi)\text{"} \quad b \approx 0$

ϕ is concave: "$\phi'(b) = l^{(b)}(\xi)$ non-increasing"

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Existence of $l(\xi) \longleftrightarrow \phi$ differentiable at $b=0$

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Precisely:

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<u>Precisely:</u>

$$\mu(\xi) = \inf_{\alpha \geq |\xi|} \alpha\, g\left(\frac{\xi}{\alpha}\right)$$

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Existence of $l(\xi)$ $\longleftrightarrow$ ϕ differentiable at $b=0$

<u>Precisely:</u>

$$\mu(\xi) = \inf_{\alpha \geq |\xi|} \alpha\, g\left(\frac{\xi}{\alpha}\right)$$

Duality:

$$\phi(b) = \inf_{\alpha \geq |\xi|} \left\{ \alpha\, g\left(\frac{\xi}{\alpha}\right) + b\alpha \right\}$$

$\mu(\xi) \longrightarrow \mu^{(b)}(\xi) = \phi(b) \approx \mu(\xi) + b\text{``}l(\xi)\text{''} \quad b \approx 0$

ϕ is concave: "$\phi'(b) = l^{(b)}(\xi)$ non-increasing"

Existence of $l(\xi) \longleftrightarrow \phi$ differentiable at $b=0$

<u>Precisely:</u>

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$[\underline{\lambda}^{(b)}, \overline{\lambda}^{(b)}]$ = Slopes of ϕ at b

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B completely disappears as unbounded component is removed!

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i.i.d. $t(e) \implies P\{t(e) = r_0\} < p_c$ enough

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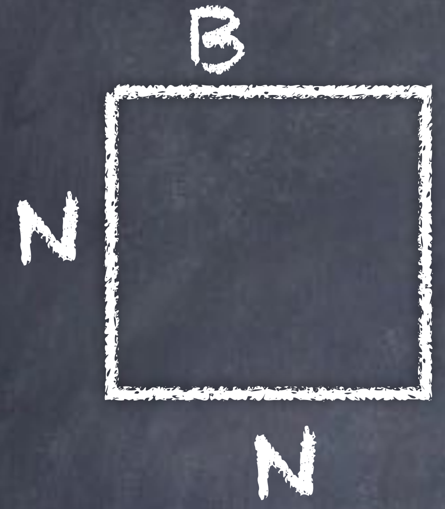

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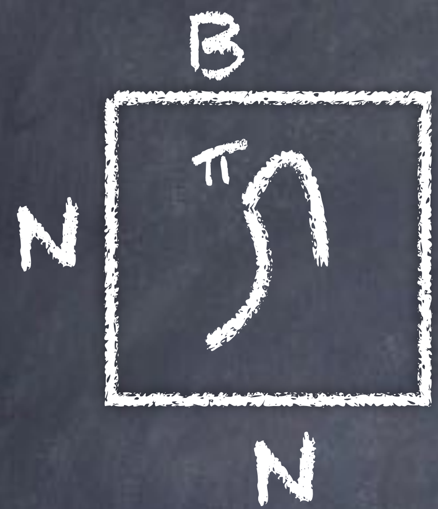

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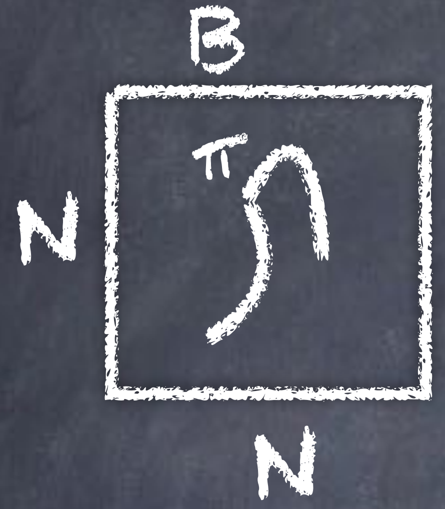

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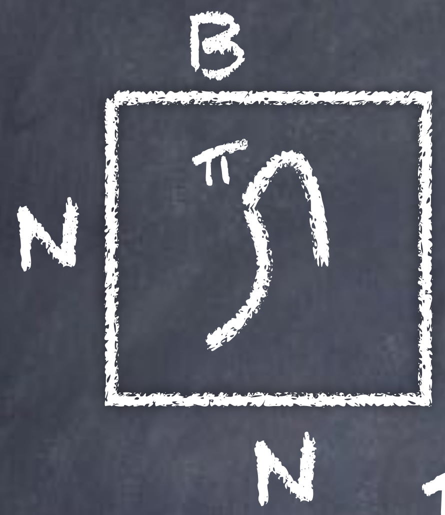

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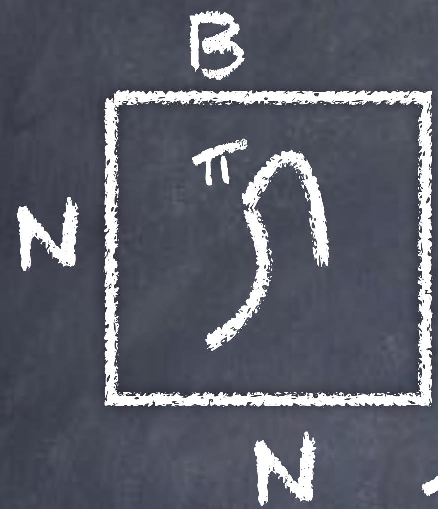

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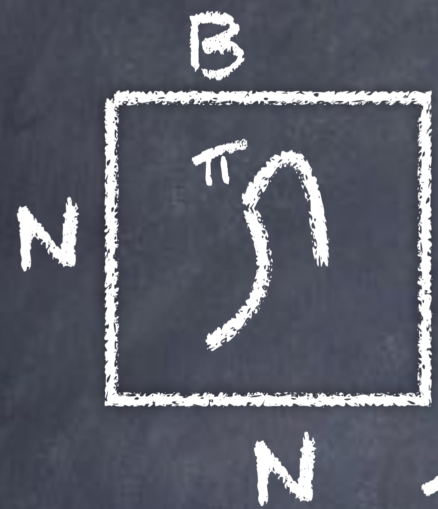

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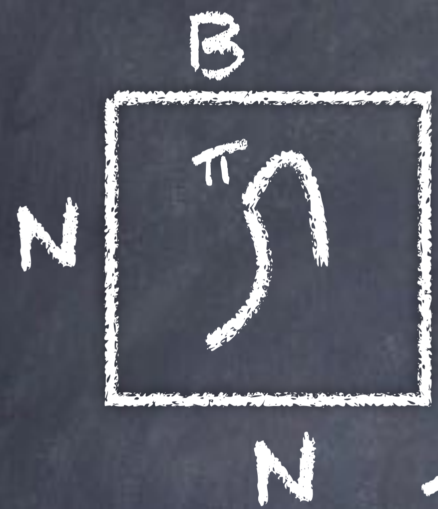

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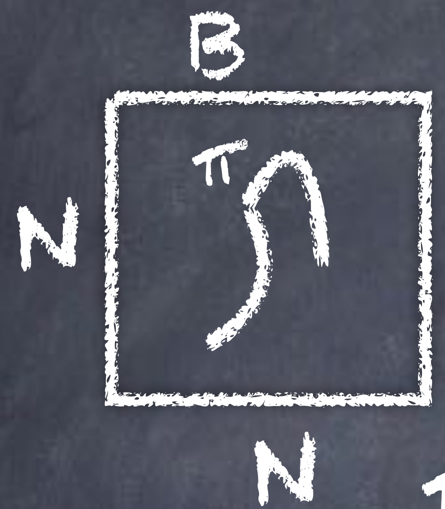

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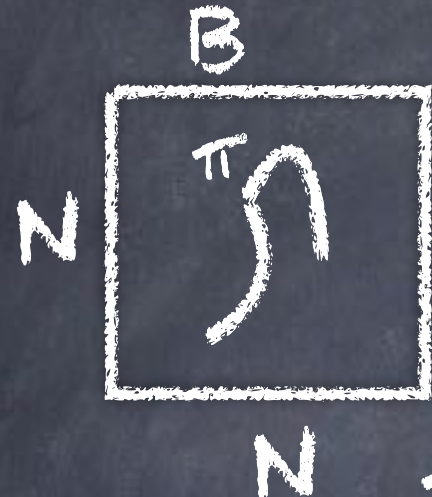

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Thank You!