FPP Geodesics

Firas Rassoul-Agha
University of Utah
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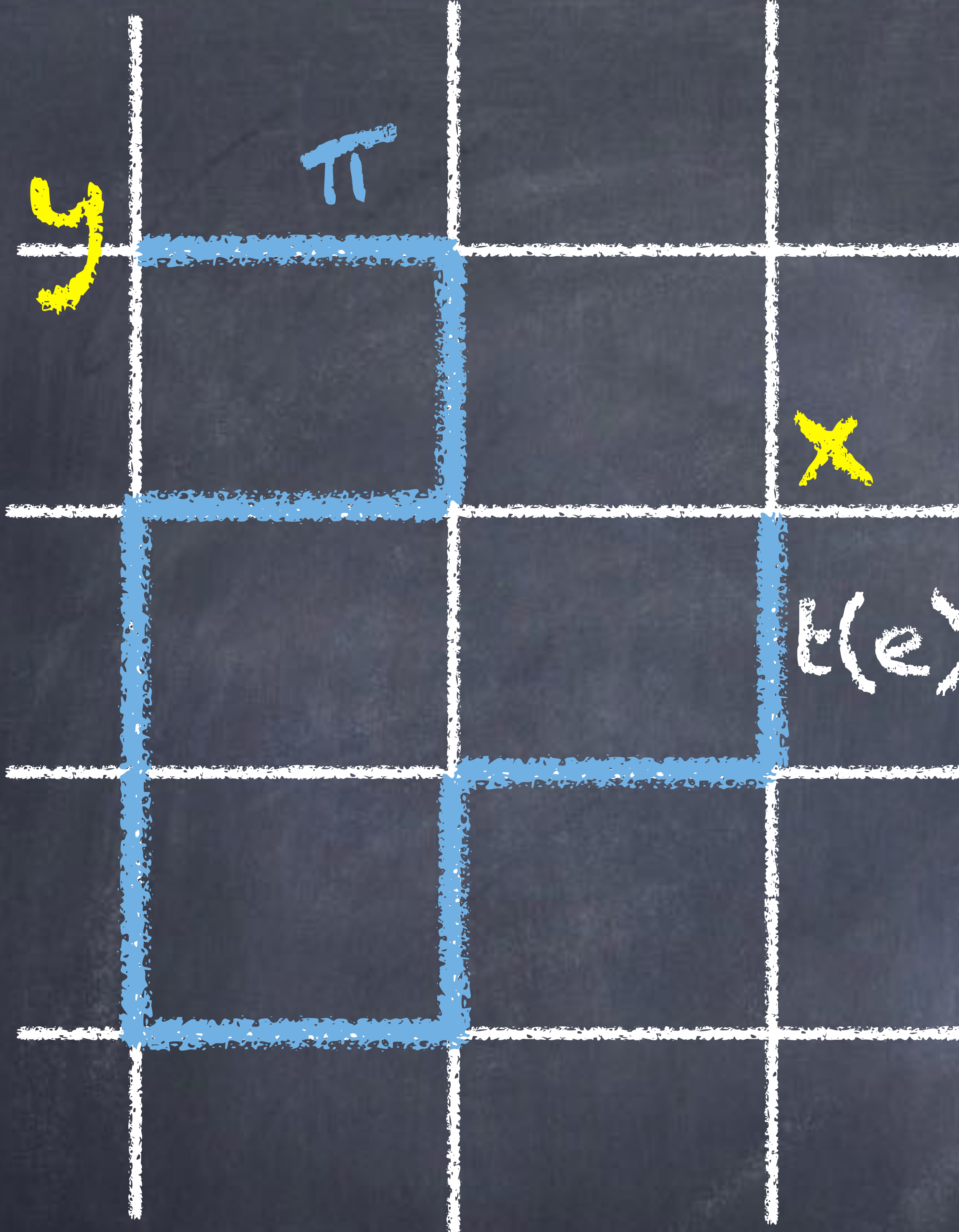
Joint with Arjun Krishnan (Rochester)
and Timo Seppäläinen (Madison)



$d \geq 2$

i.i.d. (enough moments, $>d$)

$t(e) \geq 0$

$$t(\pi) = \sum t(e)$$

$$T_{xy} = \inf_{\pi} t(\pi)$$

$$r_0 = \operatorname{ess\,inf} t(e)$$

If $r_0 = 0$: $P\{t(e)=0\} < p_c$ (bond percolation probability)

Shape Theorem (Richardson '73, Cox-Durrett '81)

$T_{0,x} \le s$ $\mu(\xi) \le 1$

μ: convex & homogenous

Norm

$\mu(\xi) > 0$ when $\xi \neq 0$

$$\overline{\lim_{|x|\to\infty}} \frac{|T_{0,x} - \mu(x)|}{|x|} = 0 \quad \text{a.s.}$$

Interested in $l(\xi)=\lim \frac{|\pi_{0,n\xi}|}{n}$ exists??

$\underline{L}_{x,y}$ = min. length

$\overline{L}_{x,y}$ = max. length

$\underline{l}(\xi)=\operatorname{essinf}\, \underline{\lim} \frac{\underline{L}_{0,n\xi}}{n}$

$\overline{l}(\xi)=\operatorname{esssup}\, \overline{\lim} \frac{\overline{L}_{0,n\xi}}{n}$

Kesten: $\overline{l}(\xi)\le C|\xi|$ (LD bounds: $P\{\overline{L}_{0,x}\ge C|x|\}\le e^{-c|x|}$)

Q. $\underline{l}(\xi)=\overline{l}(\xi)$?

$$G_{x,(n),y} = \inf_{|\pi|=n} t(\pi) \qquad G^\circ_{x,(n),y} = \inf_{|\pi|\le n} t(\pi)$$

length is now a variable

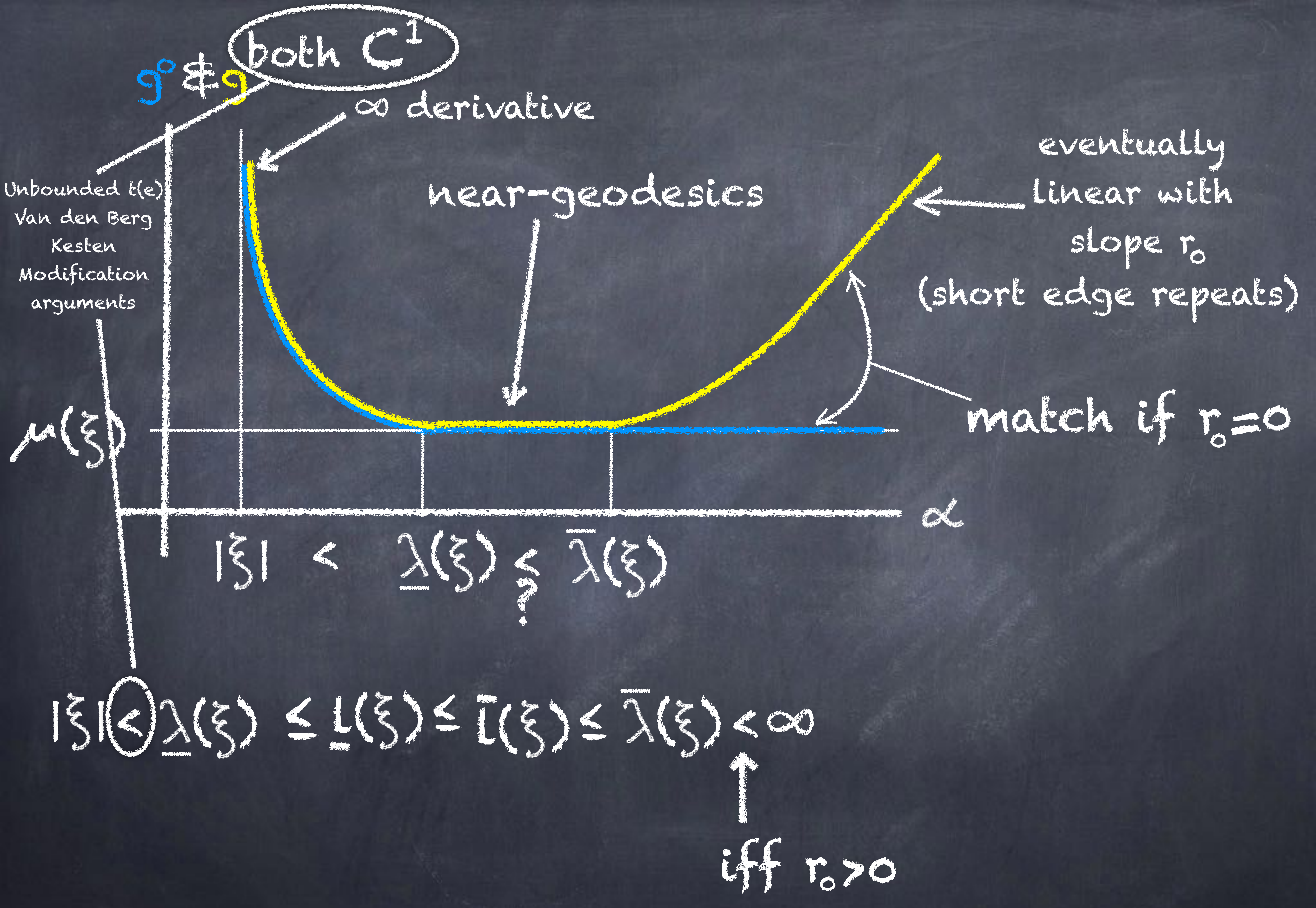
same as allowing 0 steps with $t(0)=0$

⚠ G and G° can repeat edges

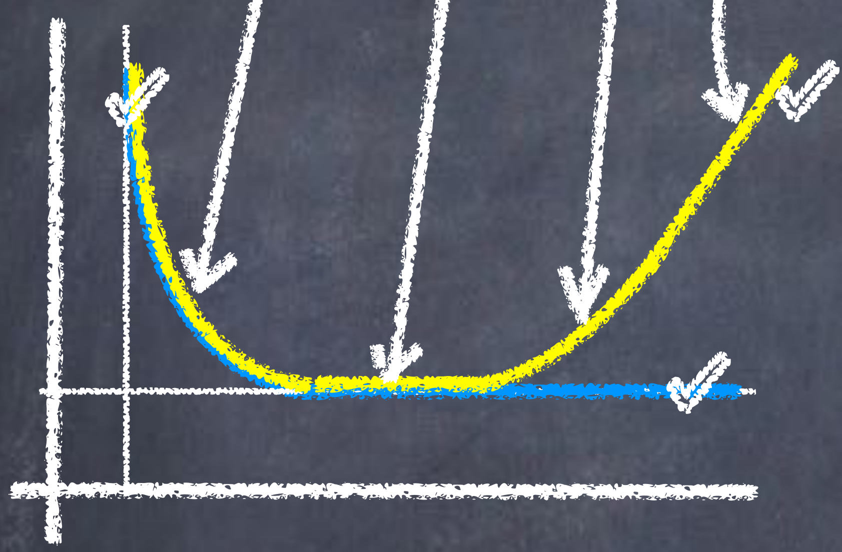
bridges G and T

$$\frac{G_{0,(n\alpha),n\xi}}{n} \longrightarrow \alpha\, g\left(\tfrac{\xi}{\alpha}\right)$$

$$\frac{G^\circ_{0,(n\alpha),n\xi}}{n} \longrightarrow \alpha\, g^\circ\left(\tfrac{\xi}{\alpha}\right)$$


both C^1
g^0 & g
∞ derivative
Unbounded t(e)
Van den Berg
Kesten
Modification
arguments
near-geodesics
eventually
linear with
slope r_0
(short edge repeats)
match if $r_0=0$
$\mu(\xi)$
α
$|\xi| < \underline{\lambda}(\xi) \overset{?}{\leq} \overline{\lambda}(\xi)$
$|\xi| < \underline{\lambda}(\xi) \leq \underline{l}(\xi) \leq \overline{l}(\xi) \leq \overline{\lambda}(\xi) < \infty$
iff $r_0>0$

Shifting $t \longrightarrow t+b = t^{(b)} \Longrightarrow \alpha g(\frac{\xi}{\alpha}) \longrightarrow \alpha g(\frac{\xi}{\alpha}) + b\alpha$



$$\overline{\lambda}^{(b)}(\xi) \xrightarrow[b\to\infty]{} |\xi|$$

(high weights $\Longrightarrow$ high sensitivity)

assume $r_0 = 0$

$$|\xi| < \underline{\lambda}^{(b)}(\xi) \le \overline{\lambda}^{(b)}(\xi) \le \underline{\lambda}^{(a)}(\xi) \le \overline{\lambda}^{(a)}(\xi) \le \underline{\lambda}(\xi) \le C|\xi| \quad (b>a)$$

Different possible geo lengths for different shifts DO NOT MIX, even for distinct typical ω!

$$\mu(\xi) \longrightarrow \mu^{(b)}(\xi) = \phi(b) \approx \mu(\xi) + b\text{``}l(\xi)\text{''} \qquad b \approx 0$$

ϕ is concave: "$\phi'(b) = l^{(b)}(\xi)$ non-increasing"

Existence of $l(\xi)$ $\longleftrightarrow$ ϕ differentiable at $b=0$

<u>Precisely:</u>

$$\mu(\xi) = \inf_{\alpha \geq |\xi|} \alpha\, g\left(\frac{\xi}{\alpha}\right)$$

Duality:

$$\phi(b) = \inf_{\alpha \geq |\xi|} \left\{ \alpha\, g\left(\frac{\xi}{\alpha}\right) + b\alpha \right\}$$

$[\underline{\lambda}^{(b)}, \overline{\lambda}^{(b)}]$ = Slopes of ϕ at b

Steele & Zhang '03: $d=2$

$t(e) \sim Ber(1-p)$ with $p < p_c$ but p close enough to p_c

Then ϕ is not differentiable at $b=0$

Conjecture: ϕ is differentiable for all $b>0$

Theorem (Krishnan-RA-Seppäläinen '18)

General $t(e)$, $r_0=0$, $0<P\{t(e)=0\}<p_c$

Then $P\{\overline{L}_{0,x} - \underline{L}_{0,x} \geq D|x|\} \geq \delta$

So ϕ is not differentiable at $b=0$

Theorem (Krishnan-RA-Seppäläinen '18)

Unbounded $t(e)$ with at least two atoms,

$r_0 \geq 0,\ 0 < P\{t(e) = r_0\} < p_c$

Then there exists a countable dense $B \subset (-r_0, \infty)$

s.t. $\forall b \in B \quad \underline{L}^{(b)}(\xi) < \overline{L}^{(b)}(\xi)$

So ϕ is not differentiable at any $b \in B$

Steel & Zhang's conjecture $\Longrightarrow$

B completely disappears as unbounded component is removed!

$\phi(b)$, $b < -r_0$?!

Replace T by

$$T^{sa} = \inf \{t(\pi) : \pi \text{ self avoiding}\}$$

OK if b is a bit $< -r_0$ (subadditivity restored by restricting to slabs)

Smythe & Wierman '78: $P\{t(e) = r_0\} < \varepsilon < p_c$

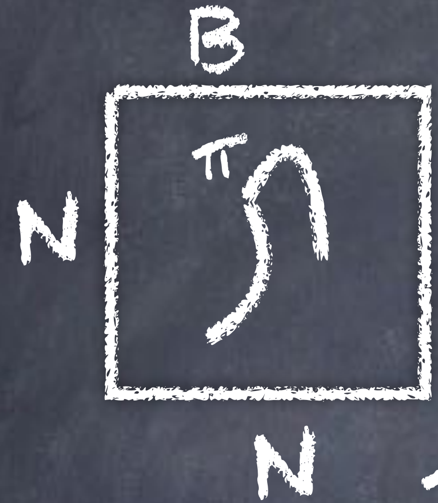
Then $\exists \delta > 0$ s.t. $b \geq -r_0 - \delta$ is OK

Works for ergodic t

Kesten '80:

i.i.d. $t(e) \Longrightarrow P\{t(e) = r_0\} < p_c$ enough

General $t(e)$, $r_0=0$, $0<P\{t(e)=0\}<p_c$



B: good if $T(\pi)>0$ $\forall \pi\subset B$ with $|\pi|\geq N$

$P\{B \text{ good}\} \xrightarrow[N\to\infty]{} 1$ (LDP)

π_{0x}: some geo from 0 to x, min. length

$\Lambda_B=\{\omega: B \text{ good}, B\cap\pi_{0x}\neq\emptyset\}$

$\sum_B P\{\Lambda_B\}=E[\#\text{good } B \text{ along } \pi_{0x}]\geq c|x|$

ω^*: resample inside B, $=\omega$ outside B

$\Delta^*_B=\{\omega^*: t^*=0 \text{ inside } B\}$ independent of Λ_B

On $\Lambda_B \cap \Delta^*_B$: every t^* geo π^*_{ox} must enter B

$$t^*(\pi^*_{ox}) = t(\pi^*_{ox}) \geq t(\pi_{ox}) > t^*(\pi_{ox})$$

π^*_{ox} w/ min. length can be made 2 steps longer

$$E[\,\underbrace{\bar{L}^*_{o,x}}_{\updownarrow\ \bar{L}_{o,x}} - \underbrace{\underline{L}^*_{o,x}}_{\updownarrow\ \underline{L}_{o,x}}\,] \geq 2\sum_B P\{\Lambda_B \cap \Delta^*_B\} = c\sum_B P\{\Lambda_B\} \geq c'|x|$$

Thank You!