

Chapter 10

Introducing Probability

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1

Idea of Probability

- ◆ Probability is the science of chance behavior: theoretical basis for statistics
- ◆ Chance behavior is unpredictable in the short run but has a regular and predictable pattern in the long run
 - this is why we can use probability to gain useful results from random samples and randomized comparative experiments

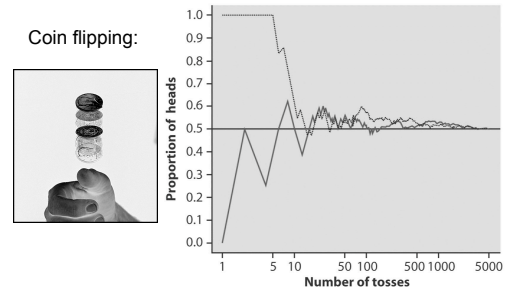
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2

Randomness and Probability

- ◆ **Random**: individual outcomes are uncertain but there is a regular distribution of outcomes in a large number of repetitions
- ◆ **Relative frequency** (proportion of occurrences) of an outcome settles down to one value over the long run. That *one value* is then defined to be the **probability** of that outcome.

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Chapter 10
3

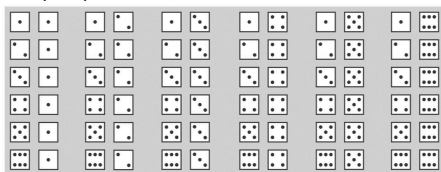
Relative-Frequency Probabilities



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4

Probability Model for Two Dice

Random phenomenon: roll pair of fair dice.
Sample space:



Probabilities: each individual outcome has probability $1/36$ (.0278) of occurring.

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5

Probability Rule 1

Any probability is a number between 0 and 1.

- ◆ A probability can be interpreted as the proportion of times that a certain event can be expected to occur.
- ◆ If the probability of an event is more than 1, then it will occur more than 100% of the time (Impossible!).

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6

Probability Rule 2

All possible outcomes together must have probability 1.

- ◆ Because some outcome must occur on every trial, the sum of the probabilities for all possible outcomes must be exactly one.
- ◆ If the sum of all of the probabilities is less than one or greater than one, then the resulting probability model will be incoherent.



Probability Rule 3

If two events have no outcomes in common, they are said to be *disjoint*. The probability that one or the other of two *disjoint* events occurs is the *sum* of their individual probabilities.

- ◆ Age of woman at first child birth
 - under 20: 25%
 - 20-24: 33%
 - 25+: ?
- } 24 or younger: 58%
- } Rules 3 and 2: 42%



Consequence

The probability that an event does not occur is 1 minus the probability that the event does occur.

- ◆ As a jury member, you assess the probability that the defendant is guilty to be 0.80. Thus you must also believe the probability the defendant is not guilty is 0.20 in order to be coherent (consistent with yourself).
- ◆ If the probability that a flight will be on time is .70, then the probability it will be late is .30.



Probability Rules: Mathematical Notation

Random phenomenon: roll pair of fair dice and count the number of pips on the up-faces.

Find the probability of rolling a 5.

$$\begin{aligned}
 P(\text{roll a 5}) &= P(\begin{array}{|c|c|} \hline 1 & 4 \\ \hline \end{array}) + P(\begin{array}{|c|c|} \hline 2 & 3 \\ \hline \end{array}) + P(\begin{array}{|c|c|} \hline 3 & 2 \\ \hline \end{array}) + P(\begin{array}{|c|c|} \hline 4 & 1 \\ \hline \end{array}) \\
 &= 1/36 + 1/36 + 1/36 + 1/36 \\
 &= 4/36 \\
 &= 0.111
 \end{aligned}$$



Normal Probability Models

- ◆ Can use density curves to assign probabilities to intervals
 - Probability outcome is between a and b equals area under density curve to the right of a and left of b
- ◆ Often Normal density curve is used



Personal Probabilities

- ◆ The degree to which a given individual believes the event in question will happen
- ◆ Personal belief or judgment
- ◆ Used to assign probabilities when it is not feasible to observe outcomes from a long series of trials
 - assigned probabilities must follow established rules of probabilities (between 0 and 1, etc.)



Personal Probabilities

◆ Examples:

- probability that an experimental (never performed) surgery will be successful
- probability that the defendant is guilty in a court case
- probability that you will receive an 'A' in this course
- probability that your favorite baseball team will win the World Series in 2020