MATH 5620 NUMERICAL ANALYSIS II
ADDENDUM TO HOMEWORK 5, DUE FRIDAY APRIL 10 2009

I hope the following comments will help you understand better the implementation of finite elements. Each comment is numbered as the question in the original HW5.

1. You do not need to derive anything. The code for constructing the stiffness matrix K is given, you just need to write it in Matlab.
2. Here you need to assemble the right hand side $F \in \mathbb{R}^{n+1}$ which depends on the right hand side f of the differential equation, more precisely:

$$F_j = (f, \phi_j) = \int_0^1 f\phi_j dx = \sum_e \int_{I_e} f\phi_j dx.$$

The idea is to do this element by element as you did for the stiffness matrix. The structure of the code would be:

```
F = zeros(n+1,1);
for e=1:n,
 F(i(e,:)) = F(i(e,:)) + Floc;
end;
```

Here `Floc` is a 2×1 vector that contains the contributions of the current element e to the right hand side F. Here are some subquestions to guide in finding `Floc` correctly. The same process appears in the class notes for triangular elements (see `http://www.math.utah.edu/~fguevara/math5620_s09/na014.pdf`, last two pages).

Note: I expect to see a derivation for `Floc`, not only code!

(a) Let $f \in V_h$. What does this mean about the restriction $f|_{I_e}$ of f to the current element I_e?

(b) Recall that on each element we have two local basis functions

$$\phi_j^e(x) = \widehat{\phi}_j\left(\frac{x - x_{i(e,1)}}{x_{i(e,2)} - x_{i(e,1)}}\right),\ j = 1, 2.$$

which have been written in terms of the basis functions at the "parent" or "reference" element $[0, 1]$: $\widehat{\phi}_1(\widehat{x}) = 1 - \widehat{x}$ and $\widehat{\phi}_2(\widehat{x}) = \widehat{x}$.

Write $f|_{I_e}(x)$ as a linear combination of $\phi_1^e(x)$ and $\phi_2^e(x)$ (you may assume that the values of f at the nodes are known).

(c) Use the local to global map $i(e, j)$ to determine for what indices i is ϕ_i non-zero on I_e. Can you express ϕ_i in terms of the local basis functions ϕ_1^e and ϕ_2^e?

(d) You now have `Floc` in terms of integrals of the kind $\int_{I_e} \phi_i^e\phi_j^e dx$, $i, j = 1, 2$. To evaluate these integrals it is more convenient to do a change of variables to go to the parent element i.e. $\widehat{x} = (x - x_{i(e,1)})/(x_{i(e,2)} - x_{i(e,1)})$.

3. This should be clear. It is a trick to hardwire the Dirichlet boundary conditions into the system $KU = F$.

4. Here you do the actual calculations for $f(x) = \exp(x)$. Obviously $f \notin V_h$, however pretend f is piecewise linear to use the code to assemble the right hand side F above (i.e. you can evaluate $f(x)$ at the nodes and run it through the code from previous question). Solve the system using Matlab's backslash "\" (do `help slash` to see the syntax). For $n = 10$ you should get a maximum error of the order of 10^{-4}. If you don't, there is something wrong with your right hand side.
5. The error should be $\mathcal{O}(h^2)$, you have to convince me that the convergence rate in your log-log plot is consistent with this.