MATH 275: Homework 4

Due Thursday, April 28

Problem 1. [A problem from Luis S.] Go to the website: http://www.math.uchicago.edu/~luis/pde/fd.html to compute the solutions to the following PDE up to time $t = 0.1$ and print out the resulting graph.

1. $u_t = u_{xx}/20$, with Dirichlet condition $u(0,t) = u(1,t) = 0$ for $t > 0$ and initial condition $u(x,t) = x\sin(5\pi x)$.
2. $u_t = u_{xx}/10 + 2u_x$, with Dirichlet condition $u(0,t) = u(1,t) = 0$ for $t > 0$ and initial condition $u(x,t) = \sin(\pi x)$.
3. $u_t = 2u_x$, with Neumann condition $u_x(0,t) = u_x(1,t) = 0$ for $t > 0$ and initial condition $u(x,t) = (1-x)\sin(3\pi x)$. (Note that the Neumann condition on the left is essentially ignored. This equation is called the transport equation.)
4. $u_t = -u_{xx}/250$, with Dirichlet condition $u(0,t) = u(1,t) = 0$ for $t > 0$ and initial condition $u(x,t) = \sin(\pi x)/5$.
5. $u_t = -u_{xx}/250$, with Dirichlet condition $u(0,t) = u(1,t) = 0$ for $t > 0$ and initial condition $u(x,t) = \sin(\pi x)/5$ plus a tiny perturbation that you draw with the mouse anywhere (just one click).

Note: A quick intro to finite difference schemes for PDE. We are approximating the solution $u(x,t)$ of a PDE on a finite interval, e.g. $[0,1]$, for times $0 \le t \le T$. We discretize space by a uniform grid of N points spaced by length h in x and M times spaced by width k in t. We will define a finite difference approximation `u[i,j]` intended to approximate the value of $u(ih, jk)$. You should discretize $u_{xx}(ih, jk) \approx$ `(u[i+1,j]+u[i-1,j]-2*u[i,j])/(h*h)`. The first derivative u_x can be discretized either as $u_x(ih, jk) \approx$ `(u[i+1,j]-u[i,j])/h` or $u_x(ih, jk) \approx$ `(u[i,j]-u[i-1,j])/h` or as $u_x(ih, jk) \approx$ `(u[i+1,j]-u[i-1,j])/(2*h)`. Depending on the particular PDE one discretization may work better than another. A typical explicit scheme to approximate the solution of a heat equation would be,

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u[i,j+1] = u[i,j] + k *(u[i+1,j]+u[i-1,j]-2*u[i,j])/(h*h)
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in this way, given the values of `u[i,j]` for $1 \le i \le N$, we could compute the values of `u[i,j+1]` and iterate. Note that in the scheme above we discretized $u_t(ih, jk) \approx$ `(u[i,j+1]-u[i,j])/k`. The word explicit means that the right hand side of the equation for `u[i,j+1]` depends only only the values of `u[.,j]` and not on `u[.,j+1]`.