

# Appendix A

## Results Cited

With which my friend Pan, heaving a great sigh, as if confessing his inability to look Infinity in the face, sank back resigned, and swallowed a large bumper of Claret.

William Makepeace Thackeray, *The Snobs of England*

We collect here several results that are cited in the text.

### A.1 Algebra and Number Theory

We need the basic idea of equivalence classes in several places. We say that  $\sim$  is an *equivalence relation* on a set  $A$  if it is *reflexive* ( $a \sim a$ ), *symmetric* ( $a \sim b$  implies  $b \sim a$ ), and *transitive* ( $a \sim b$  and  $b \sim c$  imply  $a \sim c$ ). Let us define the *equivalence class* containing  $a \in A$  by

$$[a] := \{b \in A : b \sim a\}. \tag{A.1}$$

**Theorem A.1.1.** *Given an equivalence relation  $\sim$  on a set  $A$ , every two equivalence classes  $[a]$  and  $[b]$  are either equal or mutually exclusive. Consequently, the distinct equivalence classes partition the set  $A$ .*

The proof is easy; see [Herstein](#) (1964, p. 7).

The next result is needed in the proof of Theorem 4.3.2.

**Theorem A.1.2.** *Let  $J \subset \mathbf{N}$  be closed under addition and satisfy  $\text{g.c.d.}(J) = 1$ . Then there exists a positive integer  $n_0$  such that  $n \geq n_0$  implies  $n \in J$ .*

For a proof see [Hoel, Port, and Stone](#) (1972, pp. 79–80).

The *Euler  $\varphi$ -function* is defined for each positive integer  $n$  by

$$\varphi(n) := |\{1 \leq k \leq n : \text{g.c.d.}(k, n) = 1\}|. \tag{A.2}$$

For example,  $\varphi(12) = |\{1, 5, 7, 11\}| = 4$ . The next theorem, due to Leonhard Euler [1707–1783], is needed in Corollary 9.1.3 and Problem 11.1.

**Theorem A.1.3.** *If  $m$  and  $n$  are positive integers with  $\text{g.c.d.}(m, n) = 1$ , then  $m^{\varphi(n)} \equiv 1 \pmod{n}$ .*

For a proof, see [Herstein](#) (1964, p. 37).

The next result, known as *Descartes's rule of signs*, is due to René Descartes [1596–1650]. It is needed in the proof of Lemma 9.3.3.

**Theorem A.1.4.** *The number of positive roots (counting multiplicities) of a polynomial  $p(x)$  with real coefficients is equal to the number of sign changes of the nonzero coefficients of  $p(x)$  or is less than this number by an even positive integer. The number of negative roots (counting multiplicities) of a polynomial  $p(x)$  with real coefficients is equal to the number of sign changes of the nonzero coefficients of  $p(-x)$  or is less than this number by an even positive integer.*

For example, the nonzero coefficients of  $p(x) := x^3 - x^2 - x + 1$  are 1,  $-1$ ,  $-1$ , 1, so there are two sign changes, and consequently  $p(x)$  has 2 or 0 positive roots. In addition, the nonzero coefficients of  $p(-x) = -x^3 - x^2 + x + 1$  are  $-1$ ,  $-1$ , 1, 1, so there is one sign change, hence  $p(x)$  has one negative root. In fact,  $p(x) = (x+1)(x-1)^2$ , so  $p(x)$  has two positive roots and one negative root.

For a proof, see [Albert](#) (1943).

We turn next to the cubic formula, which was first published by Girolamo [Cardano](#) [1501–1576] (1545, 1968) and is often called *Cardano's formula*. It is needed in Problems 7.12 and 13.5. Given a cubic equation

$$a_3z^3 + a_2z^2 + a_1z + a_0 = 0, \quad (\text{A.3})$$

we can assume without loss of generality that  $a_3 = 1$ .

**Theorem A.1.5.** *The three roots  $z_1, z_2, z_3$  of the cubic equation (A.3) with real coefficients and  $a_3 = 1$  can be described as follows. Let*

$$p := a_1 - \frac{a_2^2}{3}, \quad q := -a_0 + \frac{a_1a_2}{3} - \frac{2a_2^3}{27}, \quad D := \frac{p^3}{27} + \frac{q^2}{4}, \quad (\text{A.4})$$

and define

$$P := \sqrt[3]{\frac{1}{2}q + \sqrt{D}}, \quad Q := \sqrt[3]{\frac{1}{2}q - \sqrt{D}}. \quad (\text{A.5})$$

Then

$$z_1 = P + Q - \frac{a_2}{3}, \quad (\text{A.6})$$

$$z_2 = \omega P + \omega^2 Q - \frac{a_2}{3}, \quad (\text{A.7})$$

$$z_3 = \omega^2 P + \omega Q - \frac{a_2}{3}, \quad (\text{A.8})$$

where  $\omega := e^{2\pi i/3} = -\frac{1}{2} + \frac{1}{2}\sqrt{3}i$  is a cube root of unity, as is  $\omega^2 = e^{4\pi i/3} = \bar{\omega} = -\frac{1}{2} - \frac{1}{2}\sqrt{3}i$ .

*Remark.* The definitions (A.5) are slightly ambiguous because of the non-uniqueness of the cube roots. [If  $(P, Q)$  is replaced in the definitions of  $z_1, z_2,$  and  $z_3$  by  $(\omega P, \omega^2 Q)$  or by  $(\omega^2 P, \omega Q)$ , then  $z_1, z_2,$  and  $z_3$  are simply permuted.] If  $D > 0$ , then  $P$  and  $Q$  can be taken to be real and distinct, in which case  $z_1$  is real and  $z_2$  and  $z_3$  are complex conjugates; in particular,  $z_1, z_2,$  and  $z_3$  are distinct. If  $D = 0$ , then  $P$  and  $Q$  can be taken to be real and equal, in which case  $z_1, z_2,$  and  $z_3$  are real and  $z_2 = z_3$ . If  $D < 0$ , then we can take  $Q = \bar{P}$ , in which case  $z_1, z_2,$  and  $z_3$  are real and distinct. In fact, writing  $\frac{1}{2}q + \sqrt{-D}i = re^{i\theta}$ , we can take  $P = r^{1/3}e^{i\theta/3}, Q = \bar{P}$ , and

$$z_1 = 2r^{1/3} \cos(\theta/3) - \frac{a_2}{3}, \tag{A.9}$$

$$z_2 = 2r^{1/3} \cos((\theta + 2\pi)/3) - \frac{a_2}{3}, \tag{A.10}$$

$$z_3 = 2r^{1/3} \cos((\theta + 4\pi)/3) - \frac{a_2}{3}. \tag{A.11}$$

Note that  $r^{1/3} = \sqrt{-p/3}$  and  $\theta = \cos^{-1}(\frac{1}{2}q/\sqrt{-p^3/27})$ .

For a proof, see [Mac Lane and Birkhoff](#) (1988, pp. 437–438).

## A.2 Analysis and Probability

We need several results about interchanging limits, sums, integrals, and expectations. They can be stated more elegantly using measure theory because sums and expectations are special cases of abstract Lebesgue integrals, but we want to keep the presentation as elementary as possible. The results are due primarily to Henri Lebesgue [1875–1941]. The reader is referred to [Rudin](#) (1987, Chapter 1) for proofs.

We also need to define the expectation of a random variable  $X$  that is not necessarily discrete. If  $X \geq 0$ , then

$$E[X] := \int_0^\infty P(X \geq t) dt \tag{A.12}$$

works, where the right side is an improper Riemann integral. If  $X$  is arbitrary, then we can write  $X = X^+ - X^-$  and define  $E[X] := E[X^+] - E[X^-]$  as long as both expectations are finite. These definitions are consistent with the ones already given for discrete random variables (see Problem 1.32) and suffice for our purposes.

We begin with the *bounded convergence theorem*, needed in Example 1.5.10.

**Theorem A.2.1.** *Let  $X_1, X_2, \dots$  be a sequence of discrete random variables that converges a.s. to the random variable  $X$ . Assume the existence of a positive constant  $M$  such that  $|X_n| \leq M$  for all  $n \geq 1$ . Then  $\lim_{n \rightarrow \infty} E[X_n] = E[X]$ .*

The point is that we can interchange the limit and the expectation provided only that the sequence of random variables is uniformly bounded.

Next is *Fatou's lemma*, named for Pierre Fatou [1878–1929] and needed in the proofs of Theorems 3.2.2, 3.3.2, and 4.3.3.

**Lemma A.2.2.** *Let  $X_1, X_2, \dots$  be a sequence of nonnegative discrete random variables. Then  $E[\liminf_{n \rightarrow \infty} X_n] \leq \liminf_{n \rightarrow \infty} E[X_n]$ .*

This leads to the *monotone convergence theorem*, needed in the proofs of Corollaries 7.1.6 and 7.1.8 and Theorem 8.2.4.

**Theorem A.2.3.** *Let  $X_1, X_2, \dots$  be a sequence of nonnegative discrete random variables such that  $0 \leq X_1 \leq X_2 \leq \dots$ , and let  $X = \lim_{n \rightarrow \infty} X_n$ . Then  $\lim_{n \rightarrow \infty} E[X_n] = E[X]$ .*

This theorem gives the same conclusion as Theorem A.2.1 but with a monotonicity assumption instead of a uniform boundedness assumption. It yields the following corollary, needed in the proofs of Theorems 3.2.2 and 4.2.1.

**Corollary A.2.4.** *Let  $X_1, X_2, \dots$  be a sequence of nonnegative discrete random variables. Then*

$$E\left[\sum_{n=1}^{\infty} X_n\right] = \sum_{n=1}^{\infty} E[X_n]. \quad (\text{A.13})$$

Finally, the *dominated convergence theorem* generalizes the bounded convergence theorem. It is needed in the proofs of Theorems 2.1.3, 3.2.2, 4.3.2, 4.3.3, 4.4.2, 6.1.1, and 6.3.4.

**Theorem A.2.5.** *Let  $X_1, X_2, \dots$  be a sequence of discrete random variables that converges a.s. to the random variable  $X$ . Assume the existence of a random variable  $Y$  such that  $|X_n| \leq Y$  for all  $n \geq 1$  and  $E[Y] < \infty$ . Then  $\lim_{n \rightarrow \infty} E[X_n] = E[X]$ .*

The dominated convergence theorem yields the following corollary, needed in Problem 3.6.

**Corollary A.2.6.** *Let  $X_1, X_2, \dots$  be a sequence of discrete random variables such that*

$$\sum_{n=1}^{\infty} E[|X_n|] < \infty. \quad (\text{A.14})$$

Then

$$E\left[\sum_{n=1}^{\infty} X_n\right] = \sum_{n=1}^{\infty} E[X_n]. \quad (\text{A.15})$$

We also state versions of the monotone and dominated convergence theorems for Riemann integrals, needed in Problems 1.32 and 1.58.

**Theorem A.2.7.** *Let  $f_1, f_2, \dots$  be a sequence of Riemann integrable functions on an interval  $I$  such that  $0 \leq f_1 \leq f_2 \leq \dots$ , and put  $f := \lim_{n \rightarrow \infty} f_n$ . Assume that  $f$  is Riemann integrable on  $I$ . Then  $\lim_{n \rightarrow \infty} \int_I f_n(x) dx = \int_I f(x) dx$ .*

**Theorem A.2.8.** *Let  $f_1, f_2, \dots$  be a sequence of Riemann integrable functions on an interval  $I$  that converges pointwise to the Riemann integrable function  $f$  on  $I$ . Assume the existence of a Riemann integrable function  $g$  on  $I$  such that  $|f_n| \leq g$  for all  $n \geq 1$  and  $\int_I g(x) dx < \infty$ . Then  $\lim_{n \rightarrow \infty} \int_I f_n(x) dx = \int_I f(x) dx$ .*

The *Karush–Kuhn–Tucker theorem* extends the method of Lagrange multipliers to optimization problems with inequality constraints as well as equality constraints. It was first proved by William [Karush](#) [1917–1997] (1939) in a masters thesis. It was rediscovered by Harold W. Kuhn [1925–] and A. W. Tucker [1905–1995] (1951) and is often called the Kuhn–Tucker theorem. We need it in Example 10.2.3.

**Theorem A.2.9.** *Let  $f : \mathbf{R}^n \mapsto \mathbf{R}$ ,  $g_i : \mathbf{R}^n \mapsto \mathbf{R}$  for  $i = 1, \dots, k$ , and  $h_j : \mathbf{R}^n \mapsto \mathbf{R}$  for  $j = 1, \dots, l$ . Consider the problem of maximizing  $f(\mathbf{x})$ , subject to the constraints*

$$g_i(\mathbf{x}) \geq 0, \quad i = 1, \dots, k, \quad h_j(\mathbf{x}) = 0, \quad j = 1, \dots, l. \quad (\text{A.16})$$

*Suppose that  $f$ ,  $g_i$ , and  $h_j$  are continuously differentiable at  $\mathbf{x}^*$  for  $i = 1, \dots, k$  and  $j = 1, \dots, l$ , and that  $\mathbf{x}^*$  is a local maximizer of  $f$  satisfying the constraints. Assume that  $\nabla g_i(\mathbf{x}^*)$  ( $i \in \{1, \dots, k\}$  for which  $g_i(\mathbf{x}^*) = 0$ ) and  $\nabla h_j(\mathbf{x}^*)$  ( $j = 1, \dots, l$ ) are linearly independent.*

*Then there exist constants (Lagrange multipliers)  $\kappa_i \geq 0$  ( $i = 1, \dots, k$ ) and  $\lambda_j$  real ( $j = 1, \dots, l$ ) such that*

$$\nabla f(\mathbf{x}^*) + \sum_{i=1}^k \kappa_i \nabla g_i(\mathbf{x}^*) + \sum_{j=1}^l \lambda_j \nabla h_j(\mathbf{x}^*) = \mathbf{0}, \quad (\text{A.17})$$

*(A.16) holds at  $\mathbf{x} = \mathbf{x}^*$ , and  $\kappa_i g_i(\mathbf{x}^*) = 0$  for  $i = 1, \dots, k$ .*

For a proof, see [Fletcher](#) (2000, Theorem 9.1.1). The linear independence condition in the theorem is called a *constraint qualification*.

We need the negative binomial series in the definition of the negative binomial distribution in Section 1.3 and in Example 1.4.23. The proof is a straightforward calculus problem.

**Theorem A.2.10.** *For all complex  $z$  with  $|z| < 1$  and every positive integer  $n$ ,*

$$(1 - z)^{-n} = \sum_{k=0}^{\infty} \binom{k+n-1}{n-1} z^k. \quad (\text{A.18})$$

The relationship between infinite products and infinite series is needed in Example 8.1.6.

**Theorem A.2.11.** *Let  $0 < p_n < 1$  for each  $n \geq 1$ . Then*

$$\prod_{n=1}^{\infty} (1 - p_n) > 0 \quad \text{if and only if} \quad \sum_{n=1}^{\infty} p_n < \infty. \quad (\text{A.19})$$

Notice, for example, that neither condition holds when  $p_n = 1/(n+1)$  for all  $n \geq 1$ . For a proof, see [Rudin](#) (1987, p. 300).

In Section 10.3 we need the following result about the median of the binomial distribution.

**Theorem A.2.12.** *Let  $n$  be a positive integer and let  $0 < p < 1$ . Then*

$$\text{median}(\text{binomial}(n, p)) \in (np - \ln 2, np + \ln 2). \quad (\text{A.20})$$

The result is due to [Edelman](#) (c. 1979) and [Hamza](#) (1995).

The central limit theorem for samples from a finite population is needed in Section 11.3.

**Theorem A.2.13.** *For each  $N \geq 2$ , let  $(X_{N,1}, X_{N,2}, \dots, X_{N,N})$  have the discrete uniform distribution over all  $N!$  permutations of the  $N$  (not necessarily distinct but not all equal) numbers  $x_{N,1}, x_{N,2}, \dots, x_{N,N}$ , and define*

$$\mu_N := \mathbb{E}[X_{N,1}] = \frac{1}{N} \sum_{j=1}^N x_{N,j} \quad (\text{A.21})$$

and

$$\sigma_N^2 := \text{Var}(X_{N,1}) = \frac{1}{N} \sum_{j=1}^N (x_{N,j} - \mu_N)^2. \quad (\text{A.22})$$

Assume that

$$\max_{1 \leq j \leq N} \frac{|x_{N,j} - \mu_N|}{\sqrt{N\sigma_N^2}} \rightarrow 0 \quad (\text{A.23})$$

as  $N \rightarrow \infty$ . Then, with  $S_{N,n} := X_{N,1} + \dots + X_{N,n}$ , we have

$$\frac{S_{N,n} - n\mu_N}{\sqrt{n\sigma_N^2(N-n)/(N-1)}} \xrightarrow{d} N(0, 1), \quad (\text{A.24})$$

provided  $n, N \rightarrow \infty$  in such a way that  $n/N \rightarrow \alpha \in (0, 1)$ .

Notice that this generalizes Theorem 1.5.14. One of the first results of this type was proved by Erdős and Rényi (1959); see [Billingsley](#) (1968, pp. 208–212) for a generalization.

# Bibliography

Each entry is followed by a braced list of page numbers where the entry is cited. In most cases, authors' names are given as they appeared in print; see the index for full names. Occasionally, modern spellings are used (e.g., Kolmogorov instead of Kolmogoroff).

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