SEMINAR ON STOCHASTICS
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Space-Time Brownian Martingales and L^p Inequalities for ∂ and $\overline{\partial}$

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We will discuss applications of martingales to open problems concerning L^p-inequalities between ∂f and $\overline{\partial} f$ and their connections to singular integral operators. More precisely, consider the following conjecture of T. Iwaniec (1982): Let $1 < p < \infty$, and set $p^* = \max\{p, q\}$, where q is the conjugate exponent of p. Then, for $f \in W^{1,p}(\mathbb{C}, \mathbb{C})$ the following inequality should be true:

$$\|\overline{\partial} f\|_p \leq A \|\partial f\|_p, \tag{1}$$

where $A = (p^* - 1)$. This conjecture has been of considerable interest because of its many applications to quasiconformal mappings and to the regularity of solutions to some nonlinear PDEs. The said conjecture is equivalent to an estimate on the norm of the Beurling–Ahlfors operator, which is a singular integral in the complex plane.

Until recently, the best-known estimate has been that (1) holds with $A = 4(p^* - 1)$. This was discovered by the speaker and G. Wang in 1995, using martingale inequalities and stochastic integration. Recently, Nazarov and Volberg improved this by showing that (1) holds with $A = 2(p^* - 1)$. In this talk we will explain how some variations of stochastic-analysis techniques used in the work with G. Wang can be use to obtain the Nazarov–Volberg estimate, and to provide some additional information on this conjecture.

Wednesday May 1; 4:10 p.m.–5:00 p.m.; LCB 222