Math 6070-1, Spring 2006, University of Utah
Project #1
Due: Friday, February 15

Our goal is to simulate a table of binomial probabilities. You are asked to work individually (i.e., *not in groups*!) on this project. You can use any reasonable computer package, but: (i) must show *all* of your code; (ii) the code must be well documented; and (iii) cannot use canned packages. In other words, if you are asked to generate a table of probabilities, you cannot run some package that does that for you.

Suppose $S_n \sim \text{Binomial}(n\,, p)$, and define $\hat{p} = S_n/n$. Define

$$\mathcal{B}_{n,p}(z) := \mathrm{P}_p\left\{\left|\frac{\hat{p}-p}{\sqrt{\hat{p}(1-\hat{p})/n}}\right| \le z\right\}.$$

1. Let $\rho_1, \rho_2, \ldots, \rho_m$ be independent, each with the same distribution as $\hat{p}$ under P_p for every p. Define

$$\hat{\mathcal{B}}_{n,p}(z\,, m) := \frac{1}{m}\sum_{j=1}^{m} \mathbf{I}\left\{\left|\frac{\rho_j - p}{\sqrt{\rho_j(1-\rho_j)/n}}\right| \le z\right\}.$$

Then prove that as $m \to \infty$,

$$\hat{\mathcal{B}}_{n,p}(z\,, m) \stackrel{\mathrm{P}_p}{\to} \mathcal{B}_{n,p}(z).$$

2. For all $p = 0.1, 0.2, 0.3, 0.4, 0.5$, and all $n = 5, 10, 15, 20$ generate $\rho_1, \ldots, \rho_m$ i.i.d. with distribution binomial$(n\,, p)$. Use this to produce a table for $\mathcal{B}_{n,p}(z)$ for $z = 1.5, 2, 2.5, 3, 3.5$. Pay attention to what happens as you vary m, and try to compute tables that have a good amount of precision.

3. Prove that $\mathcal{B}_\infty(z) = \lim_{n\to\infty} \mathcal{B}_{n,p}(z)$ exists and is independent of p. Identify $\mathcal{B}_\infty(z)$, and create a table of values for $\mathcal{B}_\infty(z)$ for $p = 0.1, 0.2, 0.3, 0.4, 0.5$, $n = 5, 10, 15, 20$, and $z = 1.5, 2, 2.5, 3, 3.5$.

4. Compare your table for $\mathcal{B}_\infty(z)$'s with your table from Item **2**. Use your table to find exact $z_{\alpha/2}$-values for $\alpha = 0.01, 0.05$. Compare your values with the $z_{\alpha/2}$-values obtained from a normal table.

5. How would you construct a table for $p = 0.6, 0.7, 0.8, 0.9$?