Math 6070-1, Spring 2006, University of Utah
Project #3
Due: Monday May 1

1. [Applied Problem] Recall the body fat data set. It can be found at (`www.math.utah.edu/~davar/math6070/S06/Projects/Proj2/bodyfat.txt`).

(a) Find the kernel density estimate based on a $N(0\,,\sigma^2)$ kernel, where σ^2 is unknown. Do this for two or three different values of bandwidths h that are chosen with theoretical considerations in mind.

(b) Find the kernel density estimate based on a double exponential kernel of the type $K(x) = (2\eta)^{-1}\exp(-|x|/\eta)$, where η is unknown. Do this for two or three different values of bandwidths h that are chosen with theoretical considerations in mind.

(c) Compare your results to the result(s) of the command "**`density`**" supplied by `R`.

2. [Theoretical Problem] Suppose $\{X_t\}_{t=1}^{\infty}$ is a stationary time series with mean $\mu := 0$ and autocovariance function γ. Choose and fix two integer times $S < T$. Suppose we wish to estimate X_T based on X_S alone. We wish to use a *linear estimator*. That is, one of the type $\alpha X_S + \beta$. Find estimates for α and β that are optimal in the sense that they minimize the mean-squared error.

3. [Theoretical Problem] Let $\{W_t\}_{t=-\infty}^{\infty}$ be a white noise process with variance σ^2. Suppose $|\phi| < 1$, and define $X_1 := W_1$, $X_2 := \phi X_1 + W_2$, $X_3 := \phi X_2 + W_3$, $\ldots$, $X_n := \phi X_{n-1} + W_n$ $\ldots$.

(a) Prove that $\{X_t\}_{t=1}^{\infty}$ is not stationary. [Hint: It is not even weakly stationary.]

(b) Prove that nonetheless $\{X_t\}_{t=1}^{\infty}$ is "asymptotically weakly stationary," in the sense that $\gamma_0(h) := \lim_{t\to\infty} \mathrm{Cov}(X_t\,, X_{t+h})$ exists for all $h \geq 0$. Compute the said limit.

(c) [Hard] What happens if $|\phi| \geq 1$?