

9.2. By definition, $E[\exp(i\alpha \cdot G^n)] = \exp(-\frac{1}{2}\alpha \cdot Q^n \alpha)$ for all $\alpha \in \mathbf{R}^k$, where $Q_{ij}^n = E[G_i^n G_j^n]$. Therefore, $\lim_n E[\exp(i\alpha \cdot G^n)] = \exp(-\frac{1}{2}\alpha \cdot Q \alpha)$. Evidently, Q is nonnegative-definite because Q^n is. Also, Q is symmetric. Therefore, $\exp(-\frac{1}{2}\alpha \cdot Q \alpha)$ is the characteristic function of a Gaussian vector with covariance matrix Q (Theorem 9.5). The convergence theorem for characteristic functions finishes the proof.

9.5. Because W is a.s. continuous, $I(t) = \int_0^t W(s) ds$ is a Riemann integral, and is continuously-differentiable in t (fundamental theorem of calculus). We can also approximate $I(t)$ as a Riemann sum:

$$I(t) = \lim_{n \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N W\left(\frac{it}{N}\right) \quad \text{a.s.}$$

Because the vector $(W(t/N), \dots, W(t))$ is Gaussian, $I(t)$ is the limit of linear combinations of Gaussians, whence it is Gaussian (Problem **9.2**).