

9.11. Let $\mathcal{W}_t = \sigma(\{W(u) - W(t)\}_{u \geq t})$, so that $\mathcal{W} = \bigcap_{t > 0} \mathcal{W}_t$. Note that \mathcal{T} is the P-completion of \mathcal{W} . Therefore, for all $A \in \mathcal{T}$ we can find $A_* \in \mathcal{W}$ such that $P(A \Delta A_*) = 0$. By the Markov property, A_* is independent of \mathcal{F}_t^0 for all t . Therefore, A_* is independent of $\bigvee_{t > 0} \mathcal{F}_t^0$. Since $A_* \in \bigvee_{t > 0} \mathcal{F}_t^0$, it is independent of itself; hence it has probability zero or one. Because $P(A) = P(A_*)$, this proves that any element of \mathcal{T} has probability zero or one, which is the first assertion. Blumenthal's zero-one law follows upon applying this result to the Brownian motion $\{t(W1/t)\}_{t > 0}$.