

9.10. The key observation is that B is independent of $W(1)$. Indeed, note that for all $0 < t_1 < \dots < t_k < 1$, $(B(t_1), \dots, B(t_k), W(1))$ is a centered normal random variable. Therefore, it suffices to show that $\mathbb{E}[B(t_i)W(1)] = 0$, but this is easy to check. Now,

$$\mathbb{E} \left[e^{i \sum_{j=1}^m u_j W(t_j)} \mid |W(1)| \leq \varepsilon \right] = \mathbb{E} \left[e^{i \sum_{j=1}^m u_j B(t_j)} \right] \mathbb{E} \left[e^{i \sum_{j=1}^m u_j W(1)} \mid |W(1)| \leq \varepsilon \right].$$

As for the second term, conditional on $|W(1)| \leq \varepsilon$,

$$\operatorname{Im} \left(e^{i \sum_{j=1}^m u_j W(1)} \right) = \sin \left(\sum_{j=1}^m u_j W(1) \right) \leq \left| \sin \left(\varepsilon \sum_{j=1}^m |u_j| \right) \right| \leq \varepsilon \sum_{j=1}^m |u_j|,$$

for all $\varepsilon \in (0, 1/\sum_{j=1}^m |u_j|)$. This is because \sin is increasing on $(0, 1)$, and $|\sin x| = \sin x \leq x$ there. This proves that

$$\operatorname{Im} \left(\mathbb{E} \left[e^{i \sum_{j=1}^m u_j W(1)} \mid |W(1)| \leq \varepsilon \right] \right) \leq \varepsilon \sum_{j=1}^m |u_j| \rightarrow 0 \quad \text{as } \varepsilon \rightarrow 0.$$

Similarly, $\operatorname{Re}(\dots) \rightarrow 1$. This proves the result.