

8.20. If X is bounded in $L^1(\mathbb{P})$ then $\mathbb{E}[X_n^+] \leq \mathbb{E}[X_n^+] + \mathbb{E}[X_n^-] = \|X_n\|_1$ is bounded. For the converse note that $\mathbb{E}[X_n^-] = \mathbb{E}[X_n^+] - \mathbb{E}[X_n] = \mathbb{E}[X_n^+] - \mathbb{E}[X_1]$ by the martingale property. This proves that $\|X_n\|_1 = 2\mathbb{E}[X_n^+] - \mathbb{E}[X_1]$ is bounded also.