8.19. Suppose we could write our submartingale X as: $X_n = M_n + Z_n$ and $X_n = M'_n + Z'_n$, where M and M' are martingales, and Z and Z' are previsible increasing processes. Because $M - M'$ defines a martingale, this proves that $Z_n - Z'_n$ defines a martingale also. That is,

$$\mathrm{E}\left[Z_n - Z'_n \,\middle|\, \mathscr{F}_{n-1}\right] = Z_{n-1} - Z'_{n-1} \qquad \text{a.s.}$$

But $Z - Z'$ is previsible. So the preceding also equals $Z_n - Z'_n$ a.s. This proves that for all $n \geq 1$, $Z_n - Z_{n-1} = Z'_n - Z'_{n-1}$ a.s. Thus, for all $m \geq 1$, $Z_m = Z_1 + \sum_{j=2}^{m}(Z_j - Z_{j-1}) = Z_1 + \sum_{j=2}^{m}(Z'_j - Z'_{j-1}) = Z_1 - Z'_1 + Z'_m$ a.s. Thus, if we insist that $Z_1 = Z'_1 = \mathrm{E}[X_1]$, as was the case in the proof of Theorem 8.20, then $Z_m = Z'_m$ a.s. for all m.