

8.17. Let $\mathcal{F}_n = \sigma(\{X_i\}_{i=1}^n)$ and $k_n = k + R + B$, so that $f_n = X_n/k_n$. The proportion of red balls after n draws is f_n a.s. Conditional on \mathcal{F}_n , f_{n+1} is equal to $(1 + X_n)/k_{n+1} = k_{n+1}^{-1} + (k_n/k_{n+1})f_n$ with probability f_n ; it is equal to $(k_{n+1}/k_n)f_n$ with probability $1 - f_n$. Thus,

$$\mathbf{E}[f_{n+1} \mid \mathcal{F}_n] = \left(\frac{1}{k_{n+1}} + \frac{k_n}{k_{n+1}} f_n \right) f_n + \left(\frac{k_n}{k_{n+1}} f_n \right) (1 - f_n) \quad \text{a.s.}$$

The preceding is equal to $f_n(1 + k_n)/k_{n+1} = f_n$, whence the martingale property of f .