

8.15. Let $\mathcal{F}_n^0 = \sigma(\{X_i\}_{i=1}^n)$ to find that a.s.,

$$\mathbf{E} \left[M_{n+1}(t) \mid \mathcal{F}_n^0 \right] = \frac{e^{tS_n} \mathbf{E} \left[e^{tX_{n+1}} \right]}{\prod_{k=1}^{n+1} h_k(t)} = M_n(t).$$

Thus, $\{M_n(t)\}_{n=1}^\infty$ is indeed a mean-one \mathcal{F}^0 -martingale. The second part follows from the first and independence.