7.3. If X is uniform-$[0,1]$, then

$$\mathrm{E}\left[e^{it(aX+b)}\right] = \int_0^1 e^{it(ax+b)}\,dx = \frac{e^{it(a+b)} - e^{itb}}{iat}.$$

If Y is uniform=$[b, a+b]$, then

$$\mathrm{E}\left[e^{itY}\right] = \int_b^{a+b} \frac{e^{ity}}{a}\,dy = \frac{e^{it(a+b)} - e^{itb}}{iat}.$$

The uniqueness theorem does the rest. Next suppose Z is uniform-$[0,1]$. Then $Z = \sum_{i=1}^{\infty} 2^{-i} Z_i$ where the Z_i's are i.i.d. with values in $\{0,1\}$ with probability $\frac{1}{2}$ each. From Problem 1.15 we know that $X = 2Z - 1$ is uniform-$[-1,1]$. But

$$X = \sum_{i=1}^{\infty} 2^{-i} 2Z_i - 1 = \sum_{i=1}^{\infty} 2^{-i}(2Z_i - 1) = \sum_{i=1}^{\infty} 2^{-i} X_i,$$

and it follows easily that the X_i's are i.i.d. taking values ± 1 with probability $\frac{1}{2}$ each.