7.22. For all $t \in \mathbf{R}$, as $r \uparrow 1$,

$$\begin{aligned}
\mathrm{E}\left[\exp\left(it\sqrt{1-r}\sum_{j=0}^{\infty} r^j X_j\right)\right] &= \prod_{j=0}^{\infty} \mathrm{E}\left[\exp\left(it\sqrt{1-r}\; r^j X_1\right)\right] \\
&\approx \prod_{j=0}^{\infty}\left(1-\frac{t^2\sigma^2(1-r)r^{2j}}{2}\right) \\
&\approx \exp\left(-\frac{t^2\sigma^2(1-r)}{2}\sum_{j=0}^{\infty} r^{2j}\right) \approx e^{-t^2\sigma^2/4}.
\end{aligned}$$

Therefore, $\sqrt{1-r}\sum_{j=0}^{\infty} r^j X_j \Rightarrow N(0,\sigma^2/2)$.