7.12. Note that $\int_a^b e^{-itx}\,dx = (e^{-ita}-e^{-itb})/(it)$. Therefore,

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\varepsilon^2 t^2}\left(\frac{e^{-ita}-e^{-itb}}{t}\right)\widehat{\mu}(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_a^b e^{-\frac{1}{2}\varepsilon^2 t^2} e^{-itx}\widehat{\mu}(t)\,dx\,dt$$
$$= \frac{1}{2\pi}\int_a^b\int_{-\infty}^{\infty} e^{-\frac{1}{2}\varepsilon^2 t^2} e^{-itx}\widehat{\mu}(t)\,dt\,dx.$$

[Fubini–Tonelli is justified because the integrand is absolutely integrable.] Now plug in $\widehat{\mu}(t) = \int_{\mathbf{R}} e^{ity}\,\mu(dy)$ and use Fubini–Tonelli again. This yields

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\varepsilon^2 t^2}\left(\frac{e^{-ita}-e^{-itb}}{t}\right)\widehat{\mu}(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_a^b\int_{-\infty}^{\infty} e^{-\frac{1}{2}\varepsilon^2 t^2} e^{-it(x-y)}\,dt\,dx\,\mu(dy)$$
$$= \int_{-\infty}^{\infty}\int_a^b \phi_\varepsilon(x-y)\,dx\,\mu(dy);$$

see (7.24) on page 96. So now we compute the inner integral directly. [Do be brave!]

$$\int_a^b \phi_\varepsilon(x-y)\,dx = \int_{a-y}^{b-y}\phi_\varepsilon(z)\,dz = \mathrm{P}\left\{a-y \le N(0,\varepsilon^2) \le b-y\right\} = \mathrm{P}\left\{\frac{a-y}{\varepsilon} \le N(0,1) \le \frac{b-y}{\varepsilon}\right\}.$$

Firstly, this is bounded (by zero and one), uniformly for all $\varepsilon > 0$. Secondly,

$$\lim_{\varepsilon\to 0}\int_a^b \phi_\varepsilon(x-y)\,dx = \int_{a-y}^{b-y}\phi_\varepsilon(z)\,dz = \begin{cases} 1 & \text{if } a<y<b, \\ \frac{1}{2} & \text{if } a=y<b \text{ or } a<y=b \\ 0 & \text{otherwise.}\end{cases}$$

The bounded convergence theorem does the rest.