6.14. By Chebyshev's inequality, for all $\varepsilon > 0$,

$$\mathrm{P}\left\{\left|\frac{S_n - n\mu}{n}\right| > \varepsilon\right\} \leq \frac{\mathrm{Var} S_n}{n^2 \varepsilon^2}.$$

Thus there exists $C > 0$ and $n_0 \geq 1$ such that for all $n \geq n_0$, the preceding probability is $\leq C\varepsilon^{-2} n^{-\delta}$. Replace n by n^k, where $k > 1/\delta$ is a fixed integer, and then use Borel-Cantelli, to find that $S_{n^k}/n^k \to \mu$. If $n^k \leq m \leq (n+1)^k$ then

$$\frac{S_m}{m} \leq \frac{S_{(n+1)^k}}{n^k} = \frac{S_{(n+1)^k}}{(n+1)^k} \frac{(n+1)^k}{n^k} \sim \mu.$$

Similarly, $S_m/m \geq \mu + o(1)$ a.s. Thus, $S_m/m \sim \mu$ a.s. Now suppose the X_i's are identically distributed as well as uncorrelated. Then, $\mathrm{Var} S_n = n \mathrm{Var} X_1 = o(n^{2-\delta})$ for some $\delta \in (0,2)$. Thus, $S_n/n \to \mathrm{E} X_1$ a.s.