

**6.13.** Fix some  $v > 0$ . Clearly,  $\liminf_n (S_n/n) \geq \liminf_n (S_n^v/n)$ , where  $S_n^v := \sum_{i=1}^n X_i^v$ , and  $X_i^v := X_i \mathbf{1}_{\{X_i \leq v\}}$ . By the law of large numbers,  $\liminf_n (S_n^v/n) = \lim_n (S_n^v/n) = E[X_1; X_1 \leq v]$ . Let  $v \uparrow \infty$  to finish.