9.23. Choose and fix an integer $k \geq 1$, and define $T := \inf\{n \geq 1 : X_n = k\}$.

$$\mathrm{P}\left\{\max_{1\leq j\leq n} S_j \geq k\right\} = \mathrm{P}\{T \leq n\} = \mathrm{P}\{T \leq n\ ,\ S_n \leq k-1\} + \mathrm{P}\{S_n \geq k\}.$$

A.s. on $\{T \leq n\}$, $S_n = k + (S_n - S_T)$. By the strong Markov property, $S_n - S_T$ is independent of $\mathcal{F}_T$. By symmetry, $S_n - S_T$ has the same distribution as $S_T - S_n = k - S_n$. Therefore,

$$\mathrm{P}\{T \leq n\ ,\ S_n \leq k-1\} = \mathrm{P}\{T \leq n\ ,\ k + (k - S_n) \leq k-1\} = \mathrm{P}\{T \leq n\ ,\ S_n > k\} = \mathrm{P}\{S_n \geq k+1\}.$$

Parity considerations show that $\mathrm{P}\{S_n \geq k\} = \mathrm{P}\{S_n \geq k+1\}$, and both are equal to $\mathrm{P}\{S_n \leq -k\}$ by symmetry. Therefore, $\mathrm{P}\{T \leq n\} = \mathrm{P}\{S_n \leq -k\} + \mathrm{P}\{S_n \geq k\} = \mathrm{P}\{|S_n| \geq k\}$ as desired.