9.19. Evidently,

$$\left\| \int f \, dW \right\|_2^2 = \mathrm{E}\left(\iint W(s)W(t)f'(s)f'(t)\, ds\, dt \right) = \iint \min(s\,, t) f'(s) f'(t)\, ds\, dt.$$

By symmetry and integration by parts,

$$\left\| \int f \, dW \right\|_2^2 = 2\int_0^\infty \int_s^\infty s f'(s) f'(t)\, dt\, ds = -2\int_0^\infty s f(s) f'(s)\, ds = \int_0^\infty f^2(s)\, ds = \|f\|_{L^2(m)}^2.$$

Take C_c^∞ functions f_n that converge to $f \in L^2(m)$. The preceding proves that

$$\left\| \int (f_k - f_n)\, dW \right\|_2^2 = \|f_k - f_n\|_{L^2(m)}^2\,.$$

Thus, $\{\int f_n\, dW\}_{n=1}^\infty$ is a Cauchy sequence in $L^2(\mathrm{P})$, and hence converges; call the limit $\int f\, dW$. Evidently, $\| \int f\, dW\|_2 = \|f\|_{L^2(m)}$ for all $f \in L^2(m)$.

1. We just proved that
$$\mathrm{E}\left[\left(\int (f-g)\, dW \right)^2 \right] = \int (f-g)^2\, dm.$$
The left-hand side is equal to $\|f\|_{L^2(m)}^2 + \|g\|_{L^2(m)}^2 - 2\mathrm{E}[\int fg\, dW]$. The right-hand side is equal to $\|f\|_{L^2(m)}^2 + \|g\|_{L^2(m)}^2 - 2\int fg\, dm$. Whence follows the assertion.
2. If $f \in C_c^\infty$ then $G(f) = -\int W(s) f'(s)\, ds$ is Gaussian because $\int W(s) g(s)\, ds$ can be approximated by linear combinations of $W(s)$'s. Because weak limits of Gaussians are themselves Gaussians the claim follows.
3. $\mathrm{E}[G(\phi_i)G(\phi_j)] = \int \phi_i \phi_j\, dm = 0$ unless $i = j$, in which case $\mathrm{E}[G^2(\phi_i)] = \|\phi_i\|_{L(m)}^2 = 1$. Uncorrelated Gaussian are independent, so $\{G(\phi_i)\}_{i=1}^\infty$ is an i.i.d. sequence of $N(0\,,1)$'s.