9.14. Condition on $W(a)$ and use the Markov property to find that

$$\mathrm{P}\left\{\inf_{[a,b]} W > 0\right\} = \int_0^\infty \mathrm{P}\left\{\inf_{a\le t\le b} [W(t) - W(a)] > -x\right\} \mathrm{P}\{W(a) \in dx\}$$
$$= \int_0^\infty \mathrm{P}\left\{\inf_{0\le t\le b-a} W(t) > -x\right\} \frac{\exp\left(-x^2/(2a)\right)}{\sqrt{2\pi a}}\, dx.$$

By symmetry, $\inf_{[0,b-a]} W > -x$ for $x > 0$ iff $\sup_{[0,b-a]} W < x$ iff $T_x > b-a$. Therefore, by the reflection principle,

$$\mathrm{P}\left\{\inf_{[a,b]} W > 0\right\} = \int_0^\infty \mathrm{P}\left\{|W(b-a)| < x\right\} \frac{\exp\left(-x^2/(2a)\right)}{\sqrt{2\pi a}}\, dx = \frac{1}{\pi\sqrt{a(b-a)}} \int_0^\infty \int_0^x \exp\left(-\frac{y^2}{2(b-a)} - \frac{x^2}{2a}\right) dy\, dx.$$

Change variables: $u := y/\sqrt{b-a}$, $v := x/\sqrt{a}$, and $\rho := \sqrt{a/(b-a)}$ to find that

$$\mathrm{P}\left\{\inf_{[a,b]} W > 0\right\} = \frac{1}{\pi}\int_0^\infty \int_0^{\rho v} e^{-(u^2+v^2)/2}\, du\, dv = \frac{1}{\pi}\int_0^\kappa \int_0^\infty e^{-r^2/2} r\, dr\, d\theta = \frac{\kappa}{\pi},$$

where $\kappa := \arctan\rho$. By symmetry, $\mathrm{P}\{\sup_{[a,b]} W < 0\} = \kappa/\pi$ too. Therefore, the probability that W avoids zero during $(a\,,b)$ is

$$\frac{2\kappa}{\pi} = \frac{2}{\pi}\arcsin\sqrt{\frac{a}{b}}.$$

The relation to L is that $\{L < t\}$ is the event that there is no zero in $[t\,,1]$. Thus,

$$F_L(t) := \mathrm{P}\{L < t\} = \frac{2}{\pi}\arcsin\sqrt{t} \qquad {}^\forall t \in (0\,,1).$$

This is one of Paul Lévy's *arc sine laws*.