Math 5010
Sample Midterm Three

1. Suppose that, in a particular application requiring a single battery, the mean lifetime of the battery is 4 weeks with a standard deviation of 1 week. The battery is replaced by a new one when it dies, and so on. Assume lifetimes of batteries are independent. What approximately is the probability that more than 26 replacements will have to be made in a two-year period starting at the time of installation of a new battery, and not counting that new battery as a replacement? [Hint: Use a normal approximation.]

2. A fair die is rolled repeatedly. Let X denote the number of rolls until the first 6 is rolled. Let Y denote the number of rolls until the first time an even number of dots are rolled.

   (a) Are X and Y independent? Justify your answer.

   (b) Find $P\{X = Y\}$. This is the probability that we roll 6 dots at the first instance of rolling an even number of dots.

3. A certain population is comprised of half men and half women. Let X denote the total number of men in a random independent [i.e., with replacement] sample of 10 people from this population. Compute $E[e^{\lambda X}]$ for every real number λ.

4. Suppose X has a continuous distribution with density $f(x) = c/x^3$ when $x > 1$ and $f(x) = 0$ otherwise.

   (a) Compute c.

   (b) Compute $P\{X \leq 2\}$.

   (c) Compute $E(X)$ and $\mathrm{Var}(X)$.