Math 5010
Sample Midterm Two

1. Three fair dice are rolled at random and independently from one another. Let X_j denote the number of dots rolled by die number j and $X = \max(X_1, X_2)$. Find $P\{X = k\}$ for all k.

2. The probability of being dealt a full house in a hand of poker is approximately 0.0014.

   (a) What is the probability that, in 1,000 hands of poker, you will be dealt at least 2 full houses? You may assume that the hands were dealt independently from one another.

   (b) How many hands of poker should you play [independently] so that with approximate probability 0.5 you are dealt at least 50 full houses?

3. Let N be a fixed positive integer. You select a subset of $\{1, \ldots, N\}$, all possible subsets are equally likely.

   (a) What is the probability that the number i is in the randomly-selected subset? Answer this question for every $i = 1, \ldots, N$.

   (b) Use your answer to the preceding to find $E(X)$, where X denotes the number of elements of that randomly-selected subset.

4. I choose a number at random from 1 to N, where N is a fixed nonrandom positive integer. Your task is to guess my choice.

   You proceed by asking: "Is it 1? Is it 2? ..." until you find the correct number. Let X denote the number of questions you have to ask in order to find the randomly selected number.

   (a) Compute $P\{X = k\}$ for every k.

   (b) Compute $E(X)$.