Final Exam
(Mock Test)
Math 1100-002

Answer the questions in the spaces provided on the question sheets. **Show all your work**.

Name: _____ ID: U _____

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	10	10	10	10	10	10	100
Score:											

1. Evaluate the following limits.

   (a) [5 points] $\lim_{x\to 2} \dfrac{x^3 - 4x}{x^2 - 2x}$

   (b) [5 points] $\lim_{x\to +\infty} \dfrac{6x^4 - x^2 + 1}{2x^4 + 2}$

2. [10 points] Find the points of discontinuity of the function $f(x) = \begin{cases} 2 \text{ if } x \leq -2 \\ x^2 - 2 \text{ if } -2 < x < 2 \\ 4 \text{ at } x = 2 \\ \frac{x-1}{x+1} \text{ if } x > 2 \end{cases}$

3. (a) [5 points] Find the equation of the tangent line to the curve $y = \sqrt{3x^2 + 1}$ at $x = 1$.

(b) [5 points] Find $f^{(3)}(x)$ if $f'(x) = \dfrac{x}{x+1}$.

4. [10 points] Find the local minimum, local maximum points and the horizontal point of inflection for the function $f(x) = x + \dfrac{1}{x}$.

5. Find $\dfrac{dy}{dx}$ for the following functions.

(a) [2 points] $y = \ln\left(\dfrac{\sqrt[3]{x^3-2}}{x+1}\right)^2$

(b) [4 points] $x^2y^3 = e^{x+y}$

(c) [4 points] $y = e^{-1/x^2} + e^x$

6. Evaluate the following indefinite integrals.

(a) [5 points] $\displaystyle\int \frac{x+\frac{1}{2}}{x^2+x+3}\,dx$

(b) [5 points] $\displaystyle\int \left(\frac{x}{e^{x^2}} - \frac{2}{e^{2x/3}}\right) dx$

7. Evaluate the following definite integrals.

(a) [2 points] $\displaystyle\int_0^4 \sqrt{4x+9}\,dx$

(b) [4 points] $\int_0^2 8x^2 e^{-x^3}\,dx$

(c) [4 points] $\int_1^e 3y^{-1}\,dy$

8. (a) [5 points] If $\int_1^2 f(x)\,dx = \frac{2}{3}$ and $\int_2^4 f(x)\,dx = -\frac{20}{3}$, what does $\int_1^4 f(x), dx$ equal?

(b) [5 points] If $\int_1^2 (2x - x^2)\,dx = \frac{2}{3}$, what does $\int_1^2 6(2x - x^2)\,dx$ equal?

9. (a) [5 points] Find the area under the curve $y = (x-1)^2$ bounded by lines $x = 0,\ x = 2$ and the x-axis.

(b) [5 points] Find the area between the curves $y = \sqrt{x}$ and $y = x$.

10. (a) [2 points] Find the domain of the function $z = \dfrac{4x^3y - x}{2x - y}$.

(b) [2 points] If $f(w, x, y, z) = \dfrac{wx - yz^2}{xy - wz}$, then find the value of the function at the point $(2, 3, 1, -1)$.

(c) [6 points] If $z = 4y \ln x + e^{xy}$, find z_{xyy}.

11. [10 points] **(Bonus Problem):** We define $\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to+\infty} \int_{-a}^{b} f(x)\,dx$. For example,

$$\int_{-\infty}^{-1} \frac{200}{x^3}\,dx = \lim_{a\to+\infty}\left[\int_{-a}^{-1} \frac{200}{x^3}\,dx\right]$$

$$= \lim_{a\to+\infty}\left[\frac{200}{-2}\cdot\frac{1}{x^2}|_{-a}^{-1}\right] = \lim_{a\to+\infty}\left[(-100)(1-\frac{1}{a^2})\right] = -100\,[1-0] = -100$$

- Evaluate $\int_{-\infty}^{0} \frac{2x}{(x^2+1)^2}\,dx$