Ex | Solve $\frac{dy}{dt} = cy + S$

- $y = Ae^{ct} + B$

then $y' = cAe^{ct}$ is the derivative.
Plug this and y into original equation:

$$cAe^{ct} = c[Ae^{ct} + B] + S$$
$$= cAe^{ct} + cB + S$$

$$\Rightarrow 0 = cB + S \quad \Rightarrow B = -\frac{S}{c}.$$

So we have found one of our constants.
Q: How do we find A?
A: plug $t=0$ into our y.

$$y(0) = y_0 = A\underbrace{e^0}_{1} + B \quad \Rightarrow A = y_0 - B = y_0 + \frac{S}{c}$$

This Leads us to:

$$y = [y_0 + \frac{S}{c}]e^{ct} - \frac{S}{c}$$

This is the solution to the DE $\frac{dy}{dt} = cy + S$ ✓

~ Newton's Law of cooling ~

Let's call the temperature of a body y.

The temperature of the environment is y_∞.

[If y starts at y_0, it will go to y_∞ ... (think: WHY IS THIS?)]

The rate at which this happens is proportional to $y - y_\infty$, meaning the bigger the difference, the faster heat flows.

$$\frac{dy}{dt} = k(y - y_\infty)$$

Since y_∞ has the form of $-\frac{S}{c}$ from above, we can use our previous work to solve this DE ...

$\boxed{\text{EX}}$ Suppose a roast turkey is taken from the oven when its temperature has reached 185° F. It is placed on a table in a room where the temperature is 75° F.

Q1: If the temperature is 150° F after 30 minutes, what is the temperature after 45 minutes?

SOLN: our equation is $\frac{dy(t)}{dt} = k(y(t) - 75)$

our solution is $y(t) = (y_0 - 75)e^{kt} + 75$

• First, find K: → we know 150°F at 30, so use it!

(Note: $y_0 = 185°$ F) $\frac{150 - 75}{185 - 75} = e^{K(30)}$ so $K = -.01276$

Note: negative sign implies cooling. (WHY IS THIS??)

Now we can address the original problem... what is y after 45 minutes?

Plug in everything we know to find y...

$$y = (\underset{y_0}{185} - \underset{y_\infty}{75})e^{(\underset{K}{-.01276} \cdot \underset{\text{given}}{45})} + \underset{y_\infty}{75}$$

$y = 137°$ F //