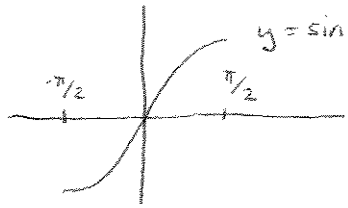
QUIZ #3 REVIEW

~ The Inverse of Trigonometric Functions ~

(§4.4)



$x = \sin^{-1}(y)$

Q: Why do we need to restrict the domain?

Recall the definition of the inverse says

$f^{-1}(x) = y \iff f(y) = x$

Q: What is the domain of $\sin^{-1}(y)$?

Q: How do we obtain the graph of $\sin^{-1}(y)$?

Q1: Find the slope of $\sin^{-1}(y)$.

ANSWER:

We calculate the derivative via
implicit differentiation (ASK YOURSELF WHY...)

By the chain rule, (slope of inverse) $= \dfrac{1}{\text{(slope of original function)}}$

$$\frac{dx}{dy} = \frac{1}{\cos(x)}$$

Since $\sin^2(x) + \cos^2(x) = 1$,

$$\cos(x) = \sqrt{1 - \sin^2(x)}$$

and since $y = \sin(x)$

$$\cos(x) = \sqrt{1 - y^2}$$

so $\dfrac{dx}{dy} = \dfrac{1}{\sqrt{1-y^2}}$ //

NOW YOU TRY:

Find the derivative of $f(y) = \cos^{-1}(y)$ and $f(y) = \tan^{-1}(y)$.

Q: How did these facts help you in Section 5.4?

(Think: Solve $\int \frac{dt}{\sqrt{1-t^2}}$.)

§5.6 ~ Properties & Applications ~

- Know some properties. We made a list in class.
- Mean Value Theorem for Integrals

-KEY- The average value from a to b is the integral divided by the length.

$$V_{avg} = \frac{1}{b-a}\int_a^b v(x)\,dx$$

(This comes from $\frac{1}{n}(v_1 + v_2 + \cdots v_n)$), with $v_1 = v(a)$ and $v_n = v(b)$

EX] Find the average of x^2 from -1 to 1.

ANS: $\frac{1}{2}\int_{-1}^{1} x^2\,dx = \frac{1}{3} = V_{avg}$ //

when does this value occur?

ANS: $x^2 = \frac{1}{3} \Rightarrow x = \pm\frac{1}{\sqrt{3}}$ //

NOW YOU TRY:

EX] Find the average value of $y = \sqrt{x}$ over the nonsymmetric interval 0 to 4.

$V_{avg} =$

Now what is the corresponding x value? (X_{avg})

Ex] (HW 5.6.32)

$0 \le x \le 1$, $0 \le y \le 1$

what is the chance that coordinates have $y^2 \le x$?

§5.7 ~ Check your homework

2] $\frac{d}{dx}\int_x^1 \cos(3t)\,dt \Rightarrow -\cos 3x$

4] $\frac{d}{dx}\int_0^2 x^n\,dt = \frac{d}{dx} x^n \int_0^2 dt = \frac{d}{dx}[x^n] t|_0^2 = 2nx^{n-1}$

16] $\frac{d}{dx}\int_{-x}^{x} \sin t\,dt = \sin(x) - (-1)\sin(-x) = 0$

because lower limit is a function so you have to do chain rule. Also, sin is an odd function, ($\sin(-x) = -\sin(x)$)

18] $\frac{d}{dx}\int_{a(x)}^{b(x)} 5\,dt = 5\,{}^{db}/_{dx} - 5\,{}^{da}/_{dx}$

20] $\frac{d}{dx}\int_0^{f(x)} \frac{df}{dt}\,dt = \frac{d}{dx}\left(f(f(x))\right) = f'(f(x))f'(x)$.

* Remember what we were doing in these problems? We were taking the derivative of an integral, and using the fundamental theorem.

§7.1 → Check your homework

* The best way to learn Integration by Parts is to practice.

2] if $e^{4x} = v$ then $v = \frac{1}{4}e^{4x}$

$uv - \int v\,du$

$(x)(\frac{1}{4})e^{4x} - \int \frac{1}{4}e^{4x}\,dx$

$= e^{4x}(x/4 - 1/16)$

28] $\int_0^1 e^{\sqrt{x}}\,dx$

let $u = x^{1/2}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$x=0 \Rightarrow u=0$

$x=1 \Rightarrow u=1$

$\Rightarrow \int_0^1 \underbrace{e^u}_{dv}\underbrace{(2u)}_{u}\,du$

$v = e^u \quad du = 2$

$uv - \int v\,du$

$(2u)(e^u) - \int 2e^u\,du$

$\Rightarrow e^u(2u-2)|_0^1 = 2$

(FROM CLASS NOTES..)
YOU TRY...

EX] $\int_0^1 \tan^{-1}(x)\,dx$