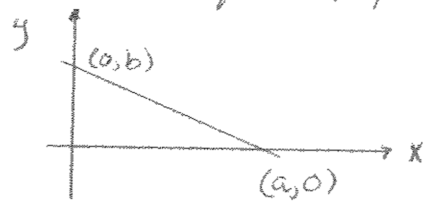
§ 1.1 Velocity & Distance

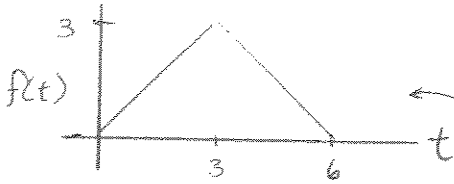
Important Concept # 1

- Find the equation for a straight line

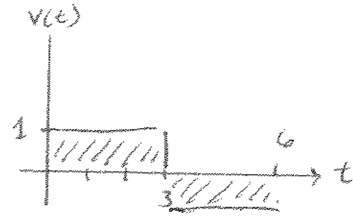


SOLN: The slope is $-b/a$, so the equation is $y = \frac{-b}{a}x - b$.

Important Concept # 2



- Draw the velocity for this graph..



⇒ Notice that the area of the velocity graph is the distance traveled. $(3+(-3)=0)$. We can see this in the $f(t)$ graph by noticing that at $t=6$, we are back to where we started.

- the equation for $f(t)$ is: $f(t) = \begin{cases} vt & \text{if } 0 \le t \le 3 \\ v(6-t) & 3 \le t \le 6. \end{cases}$

IMPORTANT:

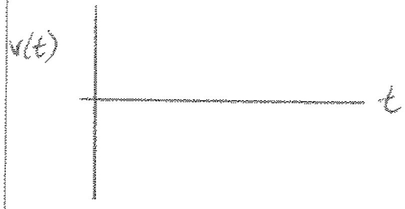
An object with a velocity changing from $+3\frac{m}{s}$ to $+9\frac{m}{s}$ is __________.

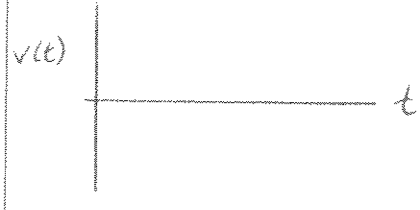
An object with a velocity changing from $-3\frac{m}{s}$ to $-9\frac{m}{s}$ is __________.

In each case, the magnitude of the velocity is __________.

DRAW A PICTURE FOR...

- speeding up in the positive direction
- speeding up in the negative direction
- slowing down in the positive direction
- slowing down in the negative direction.





Ask yourself: How would you show a change in direction based off of a velocity graph?

§1.2 Calculus without limits

Important concept # 1

- If $f = \begin{array}{|c|c|c|c|c|} \hline f_0 & f_1 & f_2 & f_3 & f_4 \\ 0 & 1 & 4 & 9 & 16 \\ \hline \end{array}$

then $v = \begin{array}{|c|c|c|c|} \hline (f_1 - f_0) & (f_2 - f_1) & (f_3 - f_2) & (f_4 - f_3) \\ 1 & 3 & 5 & 7 \\ \hline \end{array}$ (Notice the difference grows linearly...)

The notation we will use: $v_j = f_j - f_{j-1}$

$$v_1 + v_2 + v_3 + \cdots v_j = (f_1 - f_0) + (f_2 - f_1) + (f_3 - f_2) + \cdots (f_j - f_{j-1})$$

$$\Rightarrow \underline{f_j = f_0 + \sum_{n=1}^{j} v_n}$$

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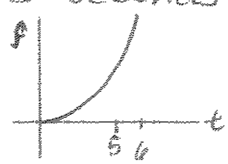
§1.3 Instantaneous Velocity

Important Concept # 1

- instantaneous velocity is the limiting value of the average velocities over shorter & shorter times...

Ex: Drop a ball 450 m above the ground. Find the ball after 5 seconds.

$$f(t) = 4.9t^2$$



* Note, I picked my coordinates such that + y is ↓.

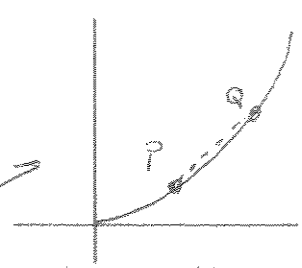
$$v_{avg} = \frac{\text{distance traveled}}{\text{time elapsed}} \rightarrow \frac{f(5.1) - f(5)}{.1}$$

time interval	average velocity
$5 \le t \le 6$	53.9
$5 \le t \le 5.1$	49.49
$5 \le t \le 5.05$	49.245
$5 \le t \le 5.01$	49.049
$5 \le t \le 5.001$	49.0049

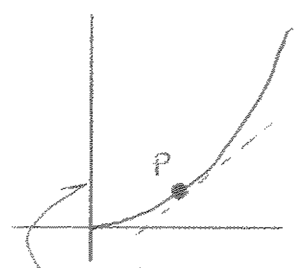
$\}$ Shortening the time period, velocity gets closer to 49 m/s.

∴ Instantaneous velocity @ 5 s is 49 $\frac{m}{s}$.

THE POINT:



The slope of the secant line = avg. velocity



The slope of the tangent line = instantaneous velocity.

§ 1.3 (continued)

IMPORTANT CONCEPT # 2

- Constant acceleration $\Rightarrow$ $a_{avg} = a_{instantaneous}$.

1) $a_{avg} = \dfrac{v - v_0}{t - t_0}$, assume $t_0 = 0$, v_0 = initial velocity

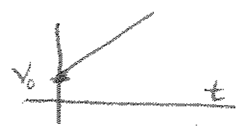
$$v = v_0 + at \qquad (\text{Note}; \ a = a_{avg} = a_{inst.})$$

2) In a similar way,

$v_{avg} = \dfrac{x - x_0}{t - t_0}$, where x_0 = initial position

$$x = x_0 + v_{avg}\, t$$

3) For Linear Velocity...

(... meaning 

) $\Rightarrow$ $v_{avg} = \frac{1}{2}(v_0 + v)$

using eqn. from 1),

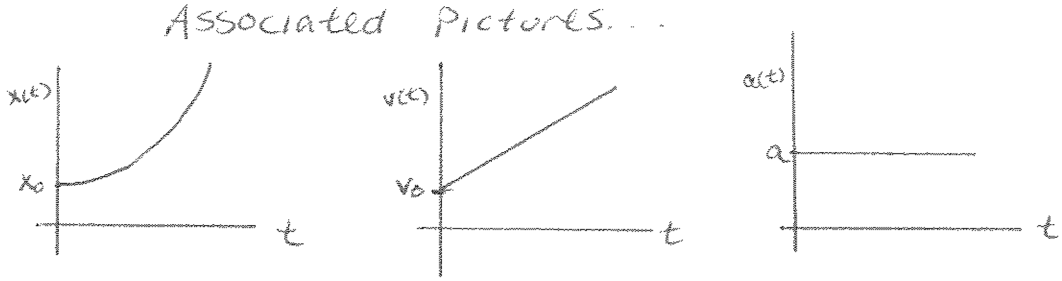
$$v_{avg} = v_0 + \tfrac{1}{2}at$$

4) We can now substitute this into eqn from 2) to get

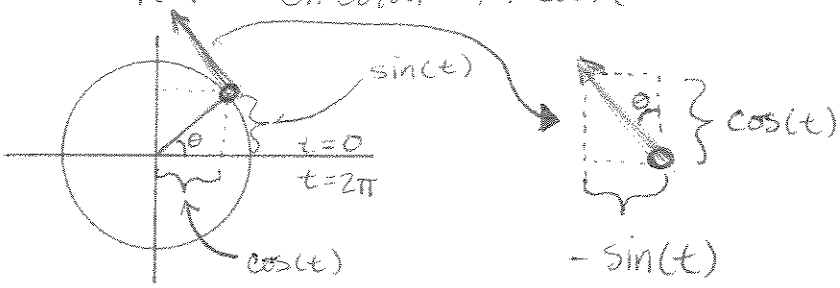
$$x = x_0 + (v_0 + \tfrac{1}{2}at)t$$

$$\Rightarrow \quad x - x_0 = v_0 t + \tfrac{1}{2}at^2 .$$

Associated Pictures...



§1.4 Circular Motion



The vertical component of the velocity
+ horizontal component of the velocity
= the vector tangent.

- The upward component of velocity is cos(t) when the upward component of position is sin(t).
- The horizontal velocity contains a minus sign. At first the ball travels to the left. The value of x is cos(t), but the speed in the x-direction is -sin t.

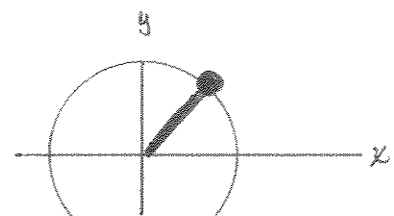
- For uniform circular motion, the <u>magnitude</u> of the velocity remains constant, but the <u>direction</u> of the velocity is continuously changing as the ball moves around the circle.

- We can assume the ball is traveling at a constant speed such that $\theta = t$.

⇒ BUT WHAT HAPPENS IF THE BALL IS GOING TWICE AS FAST?

ANS: ____________________

Write down the new (x,y) coordinates and draw the velocity vector →
for a circle of radius R.



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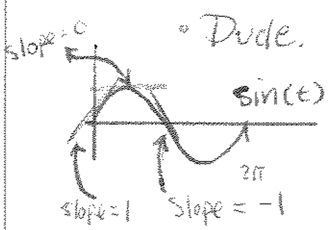
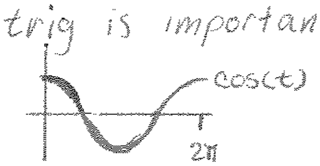
§1.6 Trig Review

- Dude. All trig is important.





- Notice the relationship between sin(t) & cos(t).

- $\sin^2(t) + \cos^2(t) = 1$
- PYTHAGORAS SAYS: $A^2 + B^2 = C^2$.
- SOH CAH TOA
- and so much more...