~ Review for Sigma Notation ~

a) $\sum_{k=1}^{n} k$ is $\underline{1+2+3\cdots n}$ b) $\sum_{j=1}^{4} (-1)^j$ is $-1+1-1+1 \underline{=0}$

c) $\sum_{j=1}^{n} 6$ is $6+6+6\cdots$ (n times) $\underline{= 6n}$

THESE SUMS DEPEND ON THE STARTING POINT AND THE STOPPING POINT.

Just a few examples.

In a), the "lower limit" is 1, the "upper limit" is n.
In b), the "lower limit" is 1, the "upper limit" is 4.

EX | How do you add the 1st 100 numbers?

Soln: $1+2+3+\cdots+100 = \sum_{j=1}^{100} j$ ← sigma notation

1 = start,
100 = stop.

Let $n = 100$

Then $1 + 100 = 101 \rightarrow 1 + n$

$n-1 = 99 \ldots \rightarrow 2 + 99 = 101 \rightarrow 2 + (n-1) = 1 + n$

Try $3 + 98 \ldots$ Notice something?

Also Notice: There are $\frac{1}{2}n$ terms if we continue in this fashion.

So $1 + 2 + \cdots (n-1) + n = \frac{1}{2}n(1+n)$

THIS MEANS..

$$\Rightarrow \sum_{j=1}^{n=100} j = \frac{1}{2}n(1+n) = \frac{1}{2}(100)(101) = \underline{5050} \;//$$

EX | Find the sum of the second hundred numbers $(101 + 102 + \cdots 200)$

Soln: We could express this as:

$$\sum_{j=101}^{200} j = \sum_{j=1}^{200} j - \sum_{j=1}^{100} j$$

then $\frac{1}{2}(200)(201) - \frac{1}{2}(100)(101) = 15{,}050.$ //

or, we could change the limits of summation.

IMPORTANT!

$$\sum_{j=101}^{200} j \text{ is the same as } \sum_{k=1}^{100} (k+100)$$

why? Because $j = k + 100$.

We just shifted things around. The sum $1+2+\cdots+100$ is 5050 as before. 100 is added to each of those terms. This gives 10,000. Then $5050 + 10000 = 15{,}050.$ //

Q1 $\sum_{i=0}^{3} 2^i = \sum_{j=1}^{4} 2^{\square}$?

Q2 $\sum_{i=3}^{n} V_i = \sum_{j=0}^{n-3} V_{\square}$?