§ 7.2

(from class discussion...)

$$\frac{1}{4}\left[\cos^3(x)\sin(x) + \frac{3}{2}x + \frac{3}{2}\sin(2x) + c\right] \stackrel{?}{=} \frac{1}{4}\left[x + \sin(2x) + \frac{x}{2} + \frac{1}{8}\sin(4x) + c\right]$$

1) $\text{¢}^3(x)\,\$(x) + \frac{3}{2}x + \frac{3}{4}\,\$(2x) \stackrel{?}{=} x + \$(2x) + \frac{x}{2} + \frac{1}{8}\,\$(4x)$

The goal is to check that the L.H.S. = R.H.S. To do this, we should change the 2x & 4x angles on the R.H.S.

culprit $\longrightarrow$ $\frac{1}{8}\,\$(4x)$

Recall: $\$(2\theta) = 2\,\$(\theta)\,\text{¢}(\theta)$

$$\Rightarrow \frac{1}{8}\,\$(4x) = \frac{1}{4}\,\$(2x)\,\text{¢}(2x)$$

1) = 2) $\text{¢}^3(x)\,\$(x) + \frac{3}{2}x + \frac{3}{2}\,\$(2x) = \frac{3}{2}x + 2\,\$(x)\text{¢}(x) + \underbrace{\frac{1}{4}\,\$(2x)\text{¢}(2x)}_{\text{change....}}$

Take down another angle... $\longrightarrow$ $\frac{1}{4}\,\$(2x)\,\text{¢}(2x)$

Recall: $\text{¢}(2\theta) = \text{¢}^2(\theta) - \$^2(\theta)$

$$\frac{1}{4}\left[2\,\$(x)\text{¢}(x)\left(\text{¢}^2(x) - \$^2(x)\right)\right]$$

$$= \frac{1}{2}\,\text{¢}^3(x)\,\$(x) - \underbrace{\frac{1}{2}\,\$^3(x)\,\text{¢}(x)}$$

$$-\frac{1}{2}\,\$^3(x)\text{¢}(x) = -\frac{1}{2}\left[1 - \text{¢}^2(x)\right]\$(x)\text{¢}(x)$$

$$= -\frac{1}{2}\,\$(x)\text{¢}(x) + \frac{1}{2}\,\text{¢}^3(x)\,\$(x)$$

$$\frac{1}{4}\,\$(2x)\,\text{¢}(2x) = \text{¢}^3(x)\,\$(x) - \frac{1}{2}\,\$(x)\text{¢}(x)$$

1) = 2) = 3) LHS $= \frac{3}{2}x + \underbrace{2\,\$(x)\text{¢}(x)} + \text{¢}^3(x)\,\$(x) - \underbrace{\frac{1}{2}\,\$(x)\text{¢}(x)}$

combine

$\frac{4}{2} - \frac{1}{2} = \frac{3}{2}$

$$= \frac{3}{2}x + \text{¢}^3(x)\,\$(x) + \frac{3}{2}\,\$(x)\text{¢}(x)$$

= RHS ☺ Dude! It totally works!

Hooray!!! (But maybe you can see a faster way?)