APPLICATION

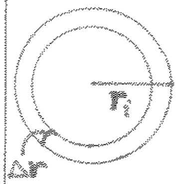
The velocity of blood that flows along a blood vessel (of radius R) can be described by:

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

l = length
r = distance from the central axis
P = pressure difference between the ends of the vessel
η = viscosity of blood.

- The "flux" is the volume of blood that passes a cross section per unit time (so units are volume per unit time).
- Notice that the velocity increases toward the center of the blood vessel. Therefore, so does the volume per unit time.
- In order to compute the flux, consider smaller, equally spaced radii $r_1, r_2 \ldots$ the area of the annulus will be:

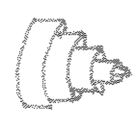
$$2\pi r_i \Delta r \qquad (\Delta r = r_i - r_{i-1})$$



If Δr is small, the velocity is almost constant throughout the annulus ($v(r_i)$), so the volume of blood per unit time that flows across the annulus is approximately

$$(2\pi r_i \Delta r)\, v(r_i)$$

And the total volume of blood flow is $\sum_{i=1}^{n} 2\pi r_i\, v(r_i)\, \Delta r$

PICTORALLY: 

As we take the limit we will get the exact value of flux

$$\text{FLUX} = \int_0^R 2\pi r\, v(r)\, dr$$

$$= \int_0^R 2\pi r \frac{P}{4\eta l}(R^2 - r^2)\, dr$$

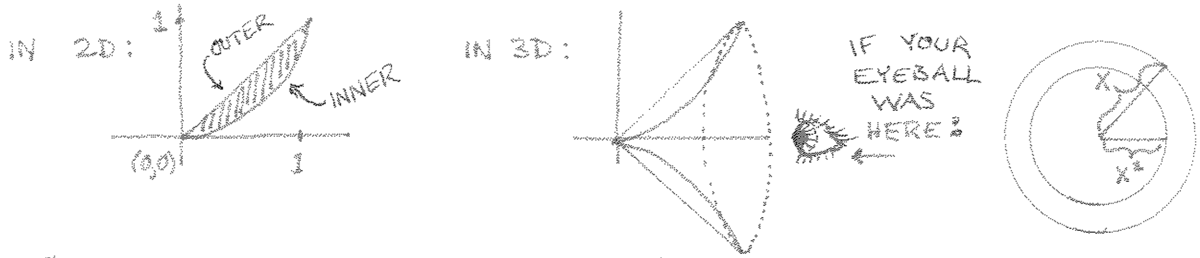
$$= \frac{\pi P}{2\eta l}\int_0^R (R^2 r - r^3)\, dr = \frac{\pi P}{2\eta l}\left[R^2\frac{r^2}{2} - \frac{r^4}{4}\right]_{r=0}^{r=R}$$

$$= \frac{\pi P}{2\eta l}\left[\frac{R^4}{2} - \frac{R^4}{4}\right] = \frac{\pi P R^4}{8\eta l}$$

This result is called "Poiseuille's Law".
It tells us that the flux is proportional to the fourth power of the radius of the blood vessel. //

Next, we'll do more practice.

$\boxed{EX}$ $y = x$ and $y = x^2$ is rotated about x-axis.



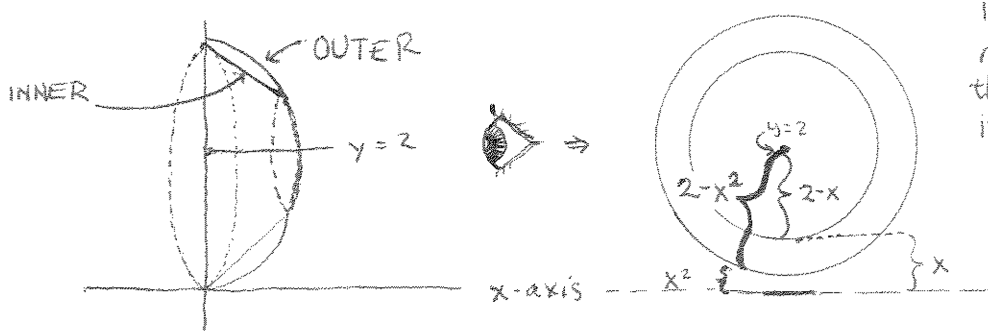
The cross-sectional area is an annulus of area

$$A(x) = \pi x^2 - \pi (x^2)^2 = \pi (x^2 - x^4)$$

Therefore,

$$V = \int_0^1 A(x)\,dx = \int_0^1 \pi (x^2 - x^4)\,dx$$
$$= \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]\Bigg|_0^1 = 2\pi / 15 \quad //$$

$\boxed{EX}$ Let's Rotate that 2D picture about the line $y = 2$.



Inner radius is now $2-x$, and the outer radius is $2-x^2$.

So now, our volume $\int_0^1 A(x)\,dx$ is:

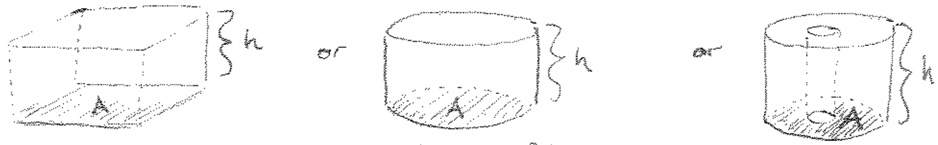
$$= \pi \int_0^1 \left[(2-x^2)^2 - (2-x)^2 \right] dx$$
$$= \pi \int_0^1 (x^4 - 5x^2 + 4x)\,dx$$
$$= \pi \left[\frac{x^5}{5} - 5\frac{x^3}{3} + 4\frac{x^2}{2} \right]\Bigg|_0^1 = \frac{8\pi}{15} \quad //$$

* Make sure you understand how to subtract (outer) - (inner) to get the desired result.

* If you were looking at horizontal slices rather than vertical slices, you would look at (right) - (left). (This corresponds to Δy slices)
(In the above problems, we did (top) - (bottom), because we were looking at Δx slices.)

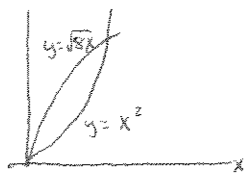
S NOT FROM BOOK | AREAS & VOLUMES | QUIZ #3 REVIEW

$$V = A \cdot h$$



$V = (lw)\cdot h$ $\quad$ $V = \pi r^2 h$ $\quad$ $V = \pi(r_2^2 - r_1^2)h$

$\boxed{EX}$ $y = x^2 \quad y = \sqrt{8x}$ about the x-axis

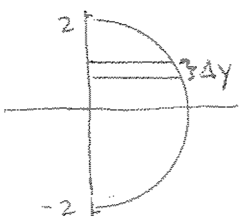


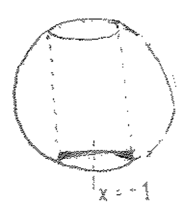
← We will get an annulus (washer)

$$V = \pi \int_0^2 (8x - x^4)\,dx$$
$$= \pi\left[\frac{8x^2}{2} - \frac{x^5}{5}\right]\Big|_0^2 = \frac{48\pi}{5} \quad //$$

$\boxed{EX}$ Semicircular region bound by $x = \sqrt{4-y^2}$ and the y-axis is revolved around $x = -1$.

- Set up the integral that represents its volume.





ANS:

$$V = \int_{-2}^{2} \pi\left[(1 + \sqrt{4-y^2})^2 - 1^2\right] dy$$

Since the part above the x-axis has the same volume as the part below it, we can integrate from 0 to 2 and double the result.

$$\star) = 2\pi \int_0^2 \left[2\sqrt{4-y^2} + 4 - y^2\right] dy$$

$\star$) Simplifies to
$$2\pi\left[\, 2\int_0^2 \sqrt{4-y^2}\,dy + \int_0^2 (4-y^2)\,dy\right.$$

Interpret the first integral as the area of a quarter of a circle (radius 2).

ANS: $\left(2\pi + \frac{16}{3}\right.$ for the 1st integral)