Ex/ $f(x) = e^{-x}$

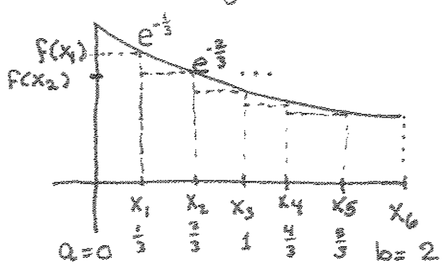
Approximate the definite integral using 6 steps between 0 and 2.

SOLUTION:

$a = 0$, $b = 2$ $\Delta x = \frac{2-0}{6} = \frac{1}{3}$ (← this is the width of rectangle.)

THING #1: Look at the graph. Find the x_i'S.

$x_1 = \frac{1}{3}$
$x_2 = \frac{2}{3}$
$x_3 = 1$
$x_4 = 1\frac{1}{3}$
$x_5 = 1\frac{2}{3}$
$x_6 = 2$



THING #2: You will evaluate the function at these points, $f(x_i)$, and use them in your SUM...

!!! Notice that the right-hand endpoints of the subintervals are

$x_1 = a + \Delta x$, $x_2 = a + 2\Delta x$, $x_3 = a + 3\Delta x$

(etc...)

The sum is going to be

$$A = e^{-x_1}\Delta x + e^{-x_2}\Delta x + e^{-x_3}\Delta x + e^{-x_4}\Delta x + e^{-x_5}\Delta x + e^{-x_6}\Delta x$$

$$= e^{-\frac{1}{3}}\left(\tfrac{1}{3}\right) + e^{-\frac{2}{3}}\left(\tfrac{1}{3}\right) + e^{-1}\left(\tfrac{1}{3}\right) + e^{-\frac{4}{3}}\left(\tfrac{1}{3}\right) + e^{-\frac{5}{3}}\left(\tfrac{1}{3}\right) + e^{-2}\left(\tfrac{1}{3}\right) \quad \leftarrow \text{EQ } (1)$$

or, using sigma notation,

$$A = \sum_{i=1}^{6} \underbrace{\left(e^{-[a+i\Delta x]}\right)}_{f(x_i)} \Delta x \quad \leftarrow \text{EQ } (2)$$

//.

Think:

What would change if you wanted to evaluate the area from $a=1$ to $b=2$, using the same stepsize? In what different ways could you re-write EQ(2), given that you are now only taking the last 3 terms of EQ(1)?

i.e. ⤳