

Chapter 5 Key Problem:

Q



$$A = \frac{1}{2} r^2 d\theta$$

• Name two reasons why that is the correct area.

• $\int_0^x A(\theta) = B$. Find x and B if you're finding the area of a quarter circle

A. Reason #1: $\frac{d\theta}{2\pi}$ is the fraction that the wedge is for the circle πr^2 .

Reason #2: It's similar to a triangle with base equal to $d\theta \cdot r$ and height equal to r . Area, A , for a triangle is $A = \frac{1}{2} b \cdot h$ so $A = \frac{1}{2} \cdot d\theta \cdot r \cdot r$
 $A = \frac{1}{2} r^2 d\theta$

$x = \frac{\pi}{2}$ for a quarter circle

$$\begin{aligned} \int_0^{\pi/2} \left(\frac{1}{2} r^2 d\theta \right) &= \frac{r^2}{2} \int_0^{\pi/2} d\theta \\ &= \frac{r^2}{2} \cdot \theta \Big|_0^{\pi/2} \\ &= \frac{\pi}{2} \cdot \frac{r^2}{2} - 0 \cdot \frac{r^2}{2} \\ &= \boxed{\frac{\pi r^2}{4}} \text{ for a quarter circle} \end{aligned}$$

R. This works because it has enough parts to quantify it for an exam, as well as making you think about the situation...

CH 5 REVIEW PROBLEM

5.5 #16

$$\int_0^1 \frac{x}{\sqrt{1-x^2}} dx = ?$$

solution

$$u = 1-x^2; \frac{du}{dx} = -2x; dx = -\frac{1}{2x} du$$

limits change: $\begin{array}{c|c} x & u \\ \hline 0 & 1 \\ 1 & 0 \end{array}$ \Rightarrow

$$-\frac{1}{2} \int_1^0 \frac{1}{\sqrt{u}} du \Rightarrow \frac{1}{2} \int_0^1 \frac{1}{\sqrt{u}} du \Rightarrow \frac{1}{2} [2u^{1/2}]_0^1 \Rightarrow \frac{1}{2} (2(1) - 2(0)) = \frac{1}{2} (2) = \boxed{1}$$

Chapter 5.5

Problems 1-10 in the exercise. I like those because they are things/skills we will actually use + apply. It is giving us building blocks to go off of.

Review Problems

SS 5.21

24)

$$dy/dx = 1/\sqrt{1-x^2}$$

$$\int dy = \int \frac{dx}{\sqrt{1-x^2}}$$

$$y = \sin^{-1} x + C$$

(4) 5 Review

Section 5.7 # 21

true or false

- a) If $\frac{d^2y}{dx^2} = \frac{dy}{dx}$ then $F(x) = S(x)$
- b) If $\frac{d^2f}{dx^2} = \frac{d^2g}{dx^2} + 4$ then $f(x) = g(x) + 4x$
- c) If $\int_a^x f(x) dx$ then the derivative of $\int_a^x f(x) dx$ is $f(x) - v(x)$
- d) the derivative of $\int_a^x v(x) dx$ is 0

- a) F
- b) F
- c) F
- d) T

- a) no guarantee that $F(x)$ and $S(x)$ have same start point
- b) no guarantee that $f(x)$ and $g(x)$ have same start
 $\frac{d^2f}{dx^2}$ & $\frac{d^2g}{dx^2}$

- c) ~~The derivative is independent of the function~~
~~that need only be given if the derivative is $\frac{d}{dx}$~~
 The derivative of an integral does not depend on function
 value.

- d) The derivative of a constant is 0.

Sometimes $\frac{d}{dx}$ starts with a derivative of how
 understanding of the material

Ch 5 problems

5.7

$$\begin{aligned} \textcircled{4} \int_0^2 x^n dx \\ &= \frac{d}{dx} 2x^{n+1} - \frac{d}{dx} 0x^{n+1} \\ &= 2nx^{n+1} \end{aligned}$$

Easy notes are happy

Chapter 5 problems

sect. 5.5 #13

$$\int_0^{\frac{\pi}{4}} \tan x \sec^2 x dx$$

$$u = \tan x \quad du = \sec^2 x dx \quad dx = \frac{du}{\sec^2 x}$$

$$\int_a^b u \sec^2 x \frac{du}{\sec^2 x} = \int_a^b u du$$

$$a = \tan(0) = 0$$

$$b = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\frac{1}{2} u^2 \Big|_0^1 = \frac{1}{2} - 0$$

$$= \boxed{\frac{1}{2}}$$

This problem employs u-substitution, trig, and computing definite integrals. This makes it a great problem that encompasses much of ch. 5.