

Optimization Problem

22 23.

$$l + w + h \leq 62''$$

$$V = l(31 - \frac{1}{2}h)^2$$

$$(62 - 2l - 2w - h)$$

$$l + w = 31 - \frac{1}{2}h$$

$$2l + 2w = 62 - h$$

$$V = (62 - 2l - 2w)h$$

$$\frac{dV}{dh} = 62h - 2lh - 2wh - h^2 = 0$$

$$\frac{2lh + 2wh}{-2h} = \frac{62h}{-2h}$$

$$l + w = \frac{62}{2} = 31$$

$$V = (62 - (2l + 2w)h)h = (62 - 2(31)h)h$$

$$V = (62 - 62h)h$$

$$\frac{dV}{dh} = (62 - 124h)$$

$$-124h + 62 = 0$$

$$\frac{1}{dh} = \frac{-3}{4}h^2 - 62h + 62 = 0$$

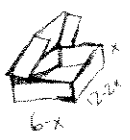
$$h = 20.8333$$

The problem took me a while
was very stressful at first,
but I was really happy
when I found the answer.
I was really proud of myself
for solving it.

Ch 3 review question

3.2

#35



Choose x to maximize volume.

Solution: $V = (6-x)(12-2x)x = (72 - 24x + 2x^2)x$

$V = 2x^3 - 24x^2 + 72x$, so $V' = 6x^2 - 48x + 72$

$V' = 6(x^2 - 8x + 12)$, $6(x-6)(x-2) = 0$
 $x = 0, 2$

@ $x=6$, $V=0$ ← minimum

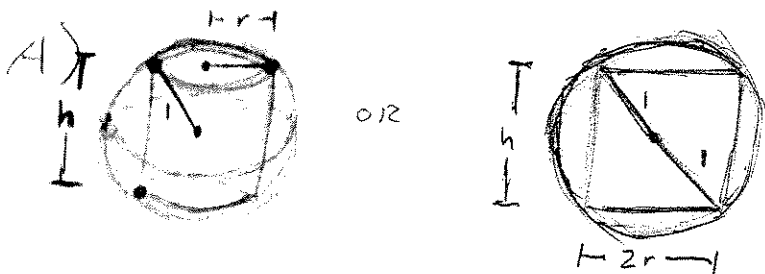
so @ $x=2$ (the max.), $V = 64$
↑
maximum

⑥

CH 3 Review

3.2 #46

Q) Find the right circular cylinder of largest volume that fits in a sphere of radius 1.



(Put h in terms of r) \Rightarrow

$$(2r)^2 + (h)^2 = (2)^2 \Rightarrow h^2 = 4 - 4r^2 \Rightarrow h = 2\sqrt{1-r^2}$$

$$V_{\text{cyl}} = \pi r^2 h \Rightarrow V = \pi r^2 (2\sqrt{1-r^2}) \Rightarrow$$

$$V = 2\pi r^2 \sqrt{1-r^2}$$

$$\frac{dV}{dr} = (2\pi r^2)' \cdot (r(1-r^2)^{1/2}) + (1-r^2)^{1/2} \cdot (4\pi r) \Rightarrow$$

$$\frac{dV}{dr} = \frac{-2\pi r^3}{\sqrt{1-r^2}} + 4\pi r \sqrt{1-r^2} \quad \text{now set } \frac{dV}{dr} \text{ equal to } 0 \text{ since}$$

we're finding a maximum \Rightarrow

$$4\pi r \sqrt{1-r^2} = \frac{2\pi r^3}{\sqrt{1-r^2}} \Rightarrow 4\pi r \cdot (1-r^2) = 2\pi r^3 \Rightarrow$$

$$4\pi r - 4\pi r^3 - 2\pi r^3 = 0 \Rightarrow 6\pi r^3 - 4\pi r = 0 \Rightarrow$$

$$r(6\pi r^2 - 4\pi) = 0 ;$$

$$r = 0$$

$$6\pi r^2 - 4\pi = 0 \Rightarrow$$

$$r^2 = \frac{4\pi}{6\pi} \Rightarrow r^2 = \frac{2}{3} \Rightarrow r = \sqrt{\frac{2}{3}}$$

(now plug $r = \sqrt{\frac{2}{3}}$ back into V) \Rightarrow

$$V = \pi \left(\sqrt{\frac{2}{3}}\right)^2 (2\sqrt{1 - \left(\frac{2}{3}\right)}) \Rightarrow \pi \left(\frac{2}{3}\right) (2\sqrt{\frac{1}{3}}) \Rightarrow$$

$$\pi \left(\frac{2}{3}\right) \left(\frac{2}{\sqrt{3}}\right) \Rightarrow \boxed{\frac{4\pi}{3\sqrt{3}}} = \text{max volume}$$

(or $\frac{4\pi\sqrt{3}}{9}$ if you want to get that square root out of the denominator)

Review chapter 3 problems:

8. Find dV of a cylinder $h=7$ and changing from $r=2$ to $r=1.9$. What if both h and r change

A. $V = \pi r^2 \cdot h$ $V = \pi r^2 \cdot h$
 $dV = \pi(2r) \cdot h \cdot dr$ or... $dV = 2\pi r h \cdot dr + \pi r^2 \cdot dh$
 $dV = 2\pi r h \cdot dr$ $dV = 2\pi r h \cdot dr + \pi r^2 \cdot dh$

whf. This problem is good in that you need to think about the best way to find dV when you have both or two changing variables. It has a real application and demands some thought process!

Review Chapter 2:

Q. Which of these functions are continuous, and why?

$$f_1(x) = \begin{cases} \sin x & x < 0 \\ \cos x & x > 0 \end{cases} \quad f_2(x) = \begin{cases} \sin \frac{1}{x} & x < 0 \\ \cos \frac{1}{x} & x > 0 \end{cases}$$

$$f_3(x) = \frac{x}{\sin x} \text{ when } \sin x \neq 0$$

$$f_4(x) = x^0 + 0x^2$$

A. $f_1(x)$ = not continuous because there's a break from 0 to 1.

$f_2(x)$ = not continuous because there's a break from 0 to 1.

$f_3(x)$ = not continuous because there's a discontinuity at $x=0$ because $\sin x \neq 0$ for two equivalent intervals around 0.

$f_4(x)$ = continuous because $f_4(0) = 1$, the limit exists and $\lim_{x \rightarrow 0} f_4(x) = 1$.

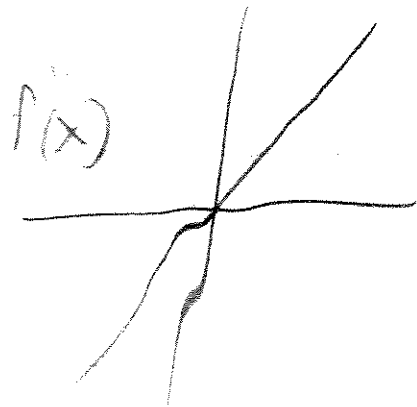
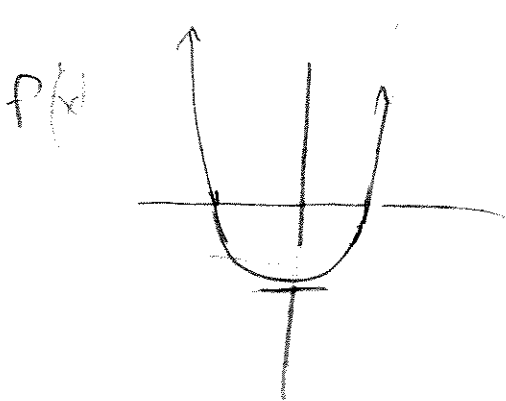
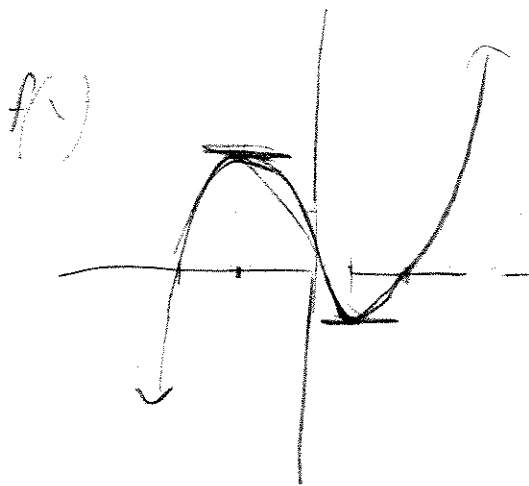
R. This was difficult because you would think about the rules of having continuous functions and then work through them multiple times which then allows you to test your skills!

Ch 3 Review Problem

Edward
Alfaro

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requires students to understand the
function and its associated derivative

for actual use, the question could
be value given function. This requires the
student to sketch the curve after
finding the critical points and then verify
local maxima and minima accordingly.

Ch. 3 Review Problem

3.8 problem 38

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$$f(b) - f(a) = f'(c)(b - a)$$

This must be positive
so the other side must be > 0

so you set up your new equation
up like $f(b) - f(a) > 0$. Then you
take $f(a)$ over to the other side $\rightarrow f(b) > f(a)$

I chose this problem because you don't have to
use numbers to solve it. It shows you that
math isn't all about numbers.

Ch 3 Best Answer Question Brayden Brasse

3.6

23 You start with a \$1000 and you spend $\frac{1}{2}$ of your money every year. Your Aunt gives you a \$1000, per year. What is x^* and what would it be for \$1000,000

$$\begin{aligned} 2x_{n+1} &= x + 1000 \\ 2x_{n+1} &= 1000 + 1000 \\ 2x_{n+1} &= \boxed{2000} \\ x_{n+1} &= 1000 \\ 2x_{n+1} &= 1000 + 1000 \\ &= \boxed{2000} \end{aligned}$$

$$\begin{aligned} 2x_{n+1} &= x + 1000 \\ 2x_{n+1} &= 1000,000 + 100 \\ x_{n+1} &= 501,000 \\ 2x_{n+1} &= \boxed{2,507,500} \\ \text{Year} &\rightarrow 2000 \end{aligned}$$

I thought it was a good problem because it applied the concept. It is also a smart board problem but it takes a second to set it up to make sense. Also without applying it is the answer 2000 while starting with 1,000,000 ~~that~~ it couldn't be normal logic.

Chapter 3 problem

3.2

$$17. \lim_{x \rightarrow \pi} \frac{x - \pi}{\sin x} \frac{d}{dx} \rightarrow \frac{1}{\cos x} \rightarrow \lim_{x \rightarrow \pi} \frac{1}{\cos x} = \boxed{-1}$$

This problem is a good application of the derivative because you need to use L'Hopital's Rule.

Chapter 3 Review Problem

3.8

#36. It has a real life situation that is applying the Mean Value theorem (MVT).

$$0 \leq x \leq 30$$