Table of Random Variables

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Name	[D]isc./[C]ont.	Notation	Interpretation	p.m.f./p.d.f.	c.d.f.	Expected Val.	Var.	m.g.f.
Bernoulli	D	$\text{Ber}(p)$	[S]uccess or [F]ailure, $\mathbb{P}(S) = p$	$p^x(1-p)^{1-x},\ x \in \{0,1\}$	$(1-p)\mathbb{1}_{[0,\infty)}(x) + p\mathbb{1}_{[1,\infty)}(x)$	p	$p(1-p)$	$1-p+pe^t,\ t \in \mathbb{R}$
Discrete Uniform	D	$\text{DUNIF}(a,b)$	Integer outcomes between a and b equally likely	$\frac{1}{b-a+1},\ x \in \{a, a+1, \ldots, b\}$	$\left(\frac{\lfloor x \rfloor - a + 1}{b-a+1}\right)\mathbb{1}_{[a,b]}(x) + \mathbb{1}_{[b+1,\infty)}(x)$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	$\frac{e^{at}-e^{(b+1)t}}{(b-a+1)\left(1-e^t\right)},\ t \in \mathbb{R}$
Binomial	D	$\text{BIN}(n,p)$	Number of S out of n i.i.d. $\text{Ber}(p)$ trials	$\binom{n}{x}p^x(1-p)^{n-x},\ x \in \{0, \ldots, n\}$	$\sum_{k=0}^{\lfloor x \rfloor}\binom{n}{k}p^k(1-p)^{n-k}$	np	$np(1-p)$	$\left(1-p+pe^t\right)^n,\ t \in \mathbb{R}$
Hypergeometric	D	$\text{HYPERGEOM}(n,M,N)$	Number of S out of n pulls w.o. replacement, M Ss, N total	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$ $x \in \{\max(0, n+M-N), \ldots, \min(n,M)\}$	$\frac{\sum_{k=0}^{\lfloor x \rfloor}\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$	$\frac{Mn}{N}$	$\frac{nM(N-M)(N-n)}{N^2(N-1)}$	$\frac{\sum_{k=0}^{n} e^{tk}\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}},\ t \in \mathbb{R}$
Geometric	D	$\text{GEOM}(p)$	Repeat i.i.d. $\text{Ber}(p)$ trials until S	$p(1-p)^{x-1}, x \in \mathbb{N}$	$\left(1-(1-p)^{\lfloor x \rfloor}\right)\mathbb{1}_{[1,\infty)}(x)$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t},\ t < -\ln(1-p)$
Negative Binomial	D	$\text{NBIN}(n,p)$	Sum of n i.i.d. $\text{GEOM}(p)$; repeat i.i.d. $\text{Ber}(p)$ trials until n Ss seen	$\binom{x}{n}p^n(1-p)^x,\ x \in \{n, n+1, \ldots\}$	$p^n\sum_{k=n}^{\lfloor x \rfloor}\binom{k}{n}(1-p)^k$	$\frac{n}{p}$	$\frac{n(1-p)}{p^2}$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^n,\ t < -\ln(1-p)$
Poisson	D	$\text{POI}(\mu)$	Number of times event occurs over some period	$\frac{e^{-\mu}\mu^x}{x!},\ x \in \{0, 1, \ldots\}$	$e^{-\mu}\sum_{k=0}^{\lfloor x \rfloor}\frac{\mu^k}{k!}$	μ	μ	$e^{\mu\left(e^t-1\right)},\ t \in \mathbb{R}$
Normal	C	$\text{N}\left(\mu,\sigma^2\right)$	Mean μ and variance σ^2	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\Phi\left(\frac{x-\mu}{\sigma}\right)$	μ	σ^2	$e^{\mu t + \frac{\sigma^2 t^2}{2}},\ t \in \mathbb{R}$
Uniform	C	$\text{UNIF}(a,b)$	Reals between a and b equally likely	$\frac{1}{b-a}\mathbb{1}_{[a,b]}(x)$	$\left(\frac{x-a}{b-a}\right)\mathbb{1}_{[a,b]}(x) + \mathbb{1}_{(b,\infty)}(x)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}\mathbb{1}_{\{t \neq 0\}}(t) + \mathbb{1}_{\{t=0\}}(t),\ t \in \mathbb{R}$
Exponential	C	$\text{EXP}(\lambda)$	Time to wait until event occurs, at rate λ per time unit	$\lambda e^{-\lambda x}\mathbb{1}_{[0,\infty)}(x)$	$\left(1-e^{-\lambda x}\right)\mathbb{1}_{[0,\infty)}(x)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t},\ t < \lambda$
Gamma	C	$\text{GAMMA}(\nu,\lambda)$	Sum of ν i.i.d. $\text{EXP}(\lambda)$ (if $\nu \in \mathbb{N}$); time to wait until event occurs ν times, at rate λ per time unit	$\frac{\lambda^\nu x^{\nu-1}e^{-\lambda x}}{\Gamma(\nu)}\mathbb{1}_{[0,\infty)}(x)$	$\begin{cases} 1-e^{-\lambda x}\sum_{k=0}^{\nu-1}\frac{(\lambda x)^k}{k!}, & \nu \in \mathbb{N} \\ \frac{\gamma(\nu,\lambda x)}{\Gamma(\nu)}, & \nu \in (0,\infty) \setminus \mathbb{N} \end{cases}$	$\frac{\nu}{\lambda}$	$\frac{\nu}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^\nu,\ t < \lambda$